Computing vertical profile of temperature in Eastern Sea using cubic spline functions

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Abstract. In this paper the spline approximation was applied to the empirical vertical profiles of oceanographic parameters such as temperature, salinity or density to obtain a more precise and reliable result of interpolation. Our experiments with the case of observed temperature profiles in Eastern Sea show that the cubic polynomial spline method has a higher reliability and precision in a comparison with the linear interpolation and other traditional methods. The method was realized as a subroutine in our programs for oceanographic data management and manipulation.

As an application, the observed temperature field from World Ocean Atlas 2001 consists of about 137000 vertical profiles have been analyzed to examine the features of the vertical distribution of temperature in Eastern Sea. It is found that the upper homogeneous layer in the summer months is only a thin one with the thickness of about 10m, but in the winter months this layer expands to the depth of about 50-60m and even more. The thickness of upper mixing layer changes largely from year to year with a range from about 20m to about 70m.

Keywords: Sea water temperature; Vertical profile of temperature; Cubic spline functions; Eastern Sea.

Temperature is always an important factor in the research of physics in general and particular in oceanography. With the rapid development of information technology, the computation and prediction of oceanographical parameters are of special interest. Sea water temperature is an important part of the input of the modern thermo-dynamical model. In many applications, the water temperature and other oceanographical parameters at different horizons are required to be calculated from their observed profiles by the interpolation procedures. The spline method of approximation appears to be a reliable and precise one for these purposes [1-4].

The cubic spline function method is aimed to find a cubic polynomial on each interval on a given coordinate line, in our case, is the z-coordinate (or depth). Suppose that on the interval [a, b] of the z-coordinate we have a computation grid $a = z_0 < z_1 < ... < z_n = b$. At each grid point, the values of the temperature function $T(z)$ at each layer where it was measured, are given by $\{T^k\}_{k=0}^{n}$. The interpolation and extrapolation problem using piece-wise cubic functions is to find a function $f(z)$ which satisfies the following conditions [5]:

- $f(z)$ belongs to $C^2(a,b)$, that is continuous together with its first and second derivatives.
- On each interval $[z_{k-1}, z_k]$, the function

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$f(z)$ is a cubic polynomial of the form:

$$f(z) = f_k(z) = \sum_{l=0}^{3} a_l^{(k)} (z - z_k)^l,\ k = 1, 2, ..., n \quad (1)$$

- Conditions at a grid point of the mesh:

$$f(z_k) = T_k,\quad k = 0, 1, ..., n \quad (2)$$

- The second derivative $f''(z)$ satisfies the conditions:

$$f''(a) = f''(b) \quad (3)$$

This problem leads to the problem of solving a system of linear equations of the coefficients $a_2^{(k)}$, $(k = 0, 1, ..., n)$:

$$h_m a_2^{(m-1)} + 2(h_m + h_{m+1}) a_2^{(m)} + h_{m+1} a_2^{(m+1)} = f(m) \quad (4)$$

$$m = 1, 2, ..., n-1$$

where: $a_2^{(0)} = 0$, $a_2^{(n)} = 0$, (5)

$$F_k = 3\left[\frac{T_{k-1} - T_k}{h_k} - \frac{T_k - T_{k+1}}{h_{k+1}}\right], \quad (6)$$

$$k = 1, 2, ..., n$$

and: $h_k = x_k - x_{k-1}$. (7)

The remaining coefficients of the system (1) are determined from the following equations:

$$a_0^{(k)} = T_k \quad (8)$$

$$a_1^{(k)} = -\frac{h_k}{3}\left(a_2^{(k-1)} + 2a_2^{(k)}\right) + \frac{T_{k-1} - T_k}{h_k} \quad (9)$$

$$a_3^{(k)} = \frac{a_3^{(k-1)} - a_2^{(k)}}{3h_k} \quad (10)$$

The solution of the problem is assumed to be exist and unique. The main difficulty in setting up the interpolation problem using spline function is to find the right boundary conditions. In the interpolation problem using data from the hydrological stations, the boundary condition (3) is quite suitable with the physical environment.

To fulfill the experiments with the spline method we use the observed profiles of water temperature in Eastern Sea in the database of World Ocean Atlas 2001. The temperature field is given for the horizons 0, 10, 20, 30, 50, 75, 100, 125, 150, 200, 250, 300, 400, 500, 600, 800 and 1000m.

Using the cubic spline functions, we computed the temperature values at different layers of distance 5m from the surface to 1000m, and the result gives us the cubic polynomials at the intervals [z_0, z_1], [z_1, z_2],..., [z_{n-1}, z_n]. For the vertical profile of temperature at the point of latitude 13°N and longitude 110°E, the computed coefficients of the polynomial for each of the 16 depth intervals are listed in Table 1. From these polynomials, we can compute the values of temperature at any layer through the system of coefficients a_0, a_1, a_2, a_3.

Table 1. Values of the coefficients of the cubic spline function at the dividing point at different depth

a_0	a_1	a_2	a_3
24.88	-0.000853	0.000128	-0.000004
24.89	-0.000014	-0.000212	0.000011
24.87	0.003910	-0.000181	-0.000001
24.87	-0.011432	0.000948	-0.000019
24.77	0.059762	-0.003820	0.000064
21.80	0.138229	0.000744	-0.000061
19.05	0.072143	0.001899	-0.000015
17.98	0.031601	-0.000278	0.000029
16.07	0.037510	0.000160	-0.000003
14.59	0.026389	0.000017	0.000001
13.34	0.023050	0.000050	0.000000
11.50	0.014124	0.000039	0.000000
10.24	0.011778	-0.000007	0.000000
9.05	0.011425	0.000011	0.000000
7.37	0.004491	0.000024	0.000000
6.72	0.001652	0.000000	0.000000

By comparing two methods, one uses the traditional linear interpolation and one uses cubic spline functions for interpolation, we can see an advantage of the latter: the cubic spline functions give smoother curve of profiles, and the profiles reflect better variation characteristics of temperature at different depths (Fig. 1).

Fig. 2 shows the computed profiles at some other points in the sea in winter period. During this time of the year, the temperature is quite low, the surface temperature is only about 24°C - 25°C.

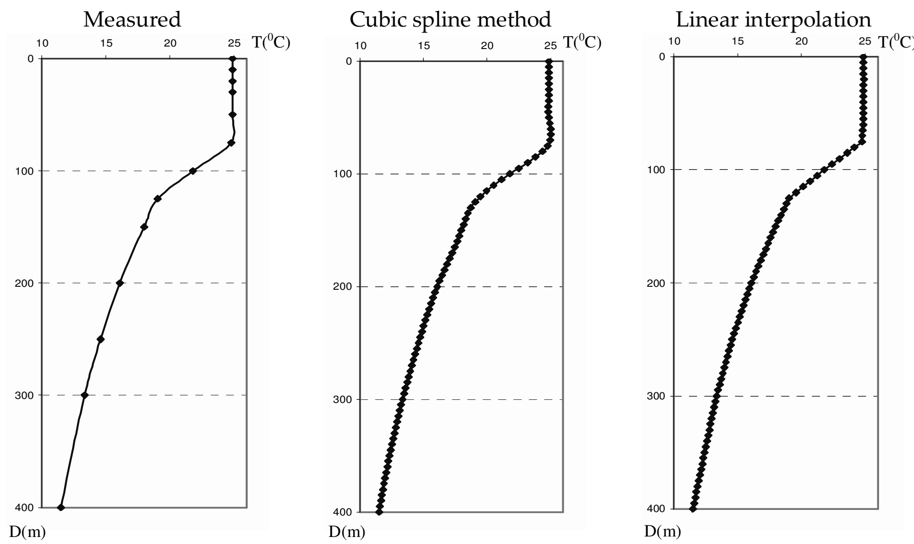

Fig. 1. Vertical distribution of temperature at point 13°N - 110°E.

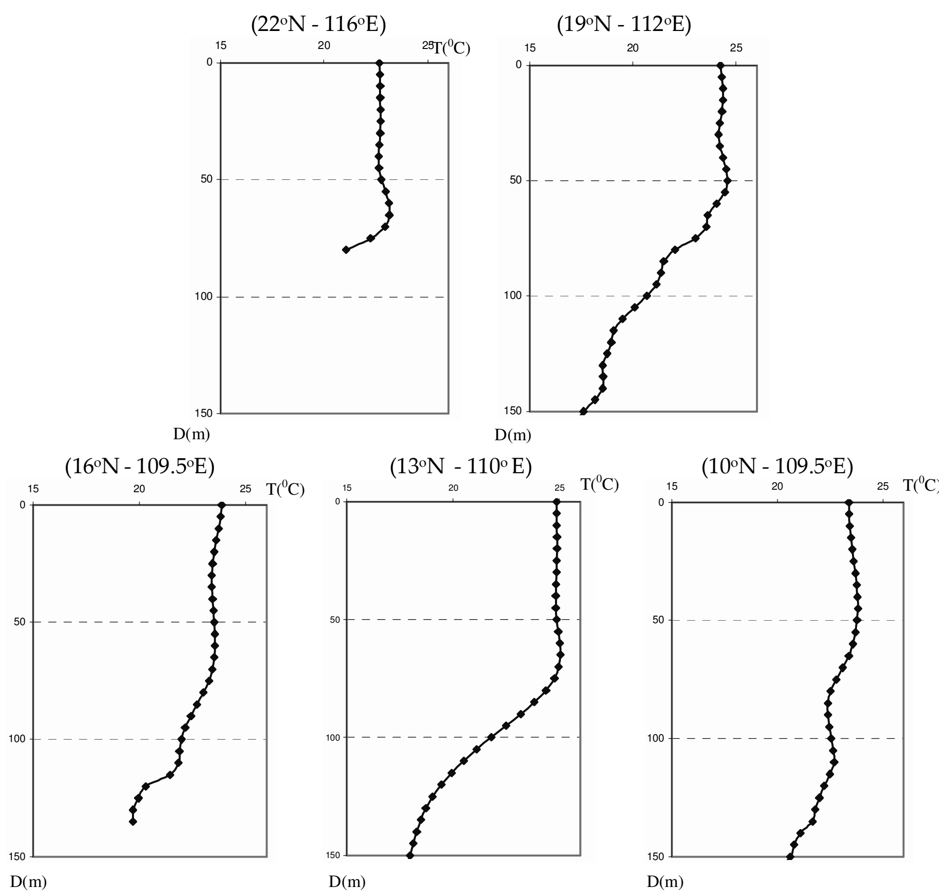

Fig. 2. Vertical distribution of temperature at various points.

Table 2. The seasonal changes of the homogeneous layer in 1966

At point 109° E - 17° N												
Month	1	2	3	4	5	6	7	8	9	10	11	12
Thickness (m)	62	60	40	10	10	15	15	–	22	50	60	60
At point 114° E - 13° N												
Month	1	2	3	4	5	6	7	8	9	10	11	12
Thickness (m)	60	65	66	45	20	–	30	30	50	40	–	–
At point 109° E - 11° N												
Month	1	2	3	4	5	6	7	8	9	10	11	12
Thickness (m)	25	–	–	–	10	8	5	–	15	30	50	–

Table 3. The changes of the winter homogeneous layer thickness between years at point 112° E - 12° N

Year	1966	1969	1972	1980	1982	1989
Thickness (m)	66	38	40	50	22	65

In general, the temperature tends to decrease as the depth increases. However, the analysis of the vertical profile of temperature at these points shows the existence of strongly mixed layers. At these points, the temperature is quite homogeneous. The strong mixing even makes it at some layers higher than the surface temperature. These points belong to the main stream area, the current speed can be as high as 1m/s at surface, so the sea water will be mixed up strongly. The thickness of this mixing layer is often about 50-70m. Under this mixing layer, there is a layer with strong variation in temperature. The temperature begins to decrease fast until 150-200m and after that it decreases gradually to the bottom. This is also the common law of changing of temperature of the sea water with depth.

Based on the analyzed vertical profiles of temperature, we can evaluate the variability of the upper homogeneous layer (Table 2). It is clear that in the summer, the upper homogeneous layer is only a thin one with the thickness of about 10m, in the winter this layer stretches to the depth of about 50-60m and even more.

The change of thickness of the homogeneous layer between years can be seen by comparison the analyzed vertical profiles at point with coordinates 112°E, 12°N in the winter of some years (Table 3).

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