Optimal adaptive sampling recovery

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Abstract: We propose an approach to study optimal methods of adaptive sampling recovery of functions by sets of a finite capacity which is measured by their cardinality or pseudo-dimension. Let W ?? L_q, 0 < q ?? ??, be a class of functions on I^d. For B a subset in L_q, we define a sampling recovery method with the free choice of sample points and recovering functions from B as follows. For each f ?? W we choose n sample points. This choice defines n sampled values. Based on these sampled values, we choose a function from B for recovering f. The choice of n sample points and a recovering function from B for each f ?? W defines a sampling recovery method S^B_n by functions in B. An efficient sampling recovery method should be adaptive to f. Given a family B of subsets in L_q, we consider optimal methods of adaptive sampling recovery of functions in W by B from B in terms of the quantity Denote $R_n(W,B)_q$ by $e_n(W)_q$ if B is the family of all subsets B of L_q such that the cardinality of B does not exceed 2^n, and by $r_n(W)_q$ if B is the family of all subsets B in L_q of pseudo-dimension at most n. Let 0 < p,q, ?? ?? ?? and ? satisfy one of the following conditions: (i) ? > d/p; (ii) ? = d/p, ?? ?? min (1,q), p,q < ??. Then for the d-variable Besov class $U^?_{p,??}$ (defined as the unit ball of the Besov space $B^?_{p,??}$), there is the following asymptotic order To construct asymptotically optimal adaptive sampling recovery methods for $e_n(U^?_{p,??})_q$ and $r_n(U^?_{p,??})_q$ we use a quasi-interpolant wavelet representation of functions in Besov spaces associated with some equivalent discrete quasi-norm. ?? 2009 Springer Science + Business Media, LLC.
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References:

1. Besov, O.V., Il'in, V.P., Nikol'skii, S.M., (1978) Integral Representations of Functions and Embedding Theorems, , Halsted Press [John Wiley & Sons], New York- Toronto, Ont.- London (1978, vol. I), (1979, vol. II), Washington D.C.: Winston & Sons
2. Birman, M.S., Solomjak, M.Z., Piecewise-polynomial approximations of the class W?p (1967) Math. USSR-Sb., 2 (3), pp. 295-317
3. Chui, C.K., (1992) An Introduction to Wavelets, , New York: Academic
4. Chui, C.K., Diamond, H., A natural formulation of quasi-interpolation by multivariate splines (1987) Proc. Am. Math. Soc., 99, pp. 643-646
5. de Boor, C., Fix, G.J., Spline approximation by quasiinterpolants (1973) J. Approx. Theory, 8, pp. 19-45
6. de Bore, C., H??llig, K., Riemenschneider, S., (1993) Box Spline, , Berlin: Springer
7. Devore, R.A., Nonlinear approximation (1998) Acta Numer., 7, pp. 51-150
8. Devore, R.A., Lorentz, G.G., (1993) Constructive Approximation, , New York: Springer
9. Devore, R.A., Popov, V.A., Interpolation of Besov spaces (1988) Trans. Am. Math. Soc., 305, pp. 397-413
10. Dung, D., On interpolation recovery for periodic functions (1991) Functional Analysis and Related Topics, pp. 224-233. , S. Koshi (Ed.), Singapore: World Scientific
11. Dung, D., On nonlinear n-widths and n-term approximation (1998) Vietnam J. Math., 26, pp. 165-176
12. Dung, D., Continuous algorithms in n-term approximation and non-linear n-widths (2000) J. Approx. Theory, 102, pp. 217-242
13. Dung, D., Asymptotic orders of optimal non-linear approximations (2001) East J. Approx., 7, pp. 55-76
14. Dung, D., Non-linear approximations using sets of finite cardinality or finite pseudo-dimension (2001) J. Complex., 17, pp. 467-492
15. D?ng, D., Non-linear sampling recovery based on quasi-interpolant wavelet representations (2009) Adv. Comput. Math., 30, pp. 375-401
16. Haussler, D., Decision theoretic generalization of the PAC model for neural net and other learning applications (1982) Inf. Comput., 100 (1), pp. 78-150
17. Haussler, D., Sphere packing number for subsets of the Boolean n-cube with bounded Vapnik-Chervonekis dimension (1995) J. Comb. Theory, Ser. A, 69, pp. 217-232
18. Kolmogorov, A.N., Tikhomirov, V.M., ?-entropy and ?-capacity of sets in function space (1959) Uspekhi Mat. Nauk, 14, pp. 3-86. , [English trans. Am. Math. Soc. Trans. 17(2), (1961)]
19. Kydryatsev, S.N., The best accuracy of reconstruction of finitely smooth functions from their values at a given number of points (1998) Izv. Math., 62, pp. 19-53
20. Nikol'skii, S., (1975) Approximation of Functions of Several Variables and Embedding Theorems, , Berlin: Springer
21. Novak, E., Deterministic and Stochastic Error Bounds in Numerical Analysis (1988) Lecture Notes in Mathematics, 1349. , Berlin: Springer
22. Novak, E., Triebel, H., Function spaces in Lipschitz domains and optimal rates of convergence for sampling (2006) Constr. Approx., 23, pp. 325-350
23. Pollard, D., Empirical processes: theory and applications (1989) NSF-CBMS Regional Conference Series in Probability and Statistics, 2. , Providence: Inst. Math., Stat. and Ann. Stat. Assoc
24. Ratsaby, J., Maiorov, V., The degree of approximation of sets in Euclidean space using sets with bounded Vapnik-

Chervonekis dimension (1998) Discrete Appl. Math., 86, pp. 81-93
25. Ratsaby, J., Maiorov, V., On the degree of approximation by manifolds of finite pseudo-dimension (1999) Constr. Approx., 15, pp. 291-300
26. Temlyakov, V., (1993) Approximation of Periodic Functions, , New York: Nova Science
27. Temlyakov, V., Nonlinear methods of approximation (2003) Fund. Comput. Math., 3, pp. 33-107
28. Vapnik, V.N., Chervonekis, A.Y., Necessary and sufficient conditions for the uniform convergence of means to their expectations (1981) Theory Probab. Appl., 26, pp. 264-280