

# Massera's theorem for almost periodic solutions of functional differential equations

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**Abstract:** The Massera Theorem for almost periodic solutions of linear periodic ordinary differential equations of the form (\*)  $x' = A(t)x + f(t)$ , where  $f$  is almost periodic, is stated and proved. Furthermore, it is extended to abstract functional differential equations (\*\*\*)  $x' = Ax + F(t)x_t + f(t)$ , where  $A$  is the generator of a compact semigroup,  $F$  is periodic and  $f$  is almost periodic. The main techniques used in the proofs involve a new variation of constants formula in the phase space and a decomposition theorem for almost periodic solutions.

**Author Keywords:** Abstract functional differential equation; Almost periodic solutions; Decomposition; Massera's theorem; Variation of constants formula

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