

Multiple solutions for a class of quasilinear elliptic equations of $p(x)$ -Laplacian type with nonlinear boundary conditions

Chung N.T., Ng Q.-A.

Department of Mathematics and Informatics, Quang Binh University, 312 Ly Thuong Kiet, Dong Hoi, Quang Binh, Viet Nam; Department of Mathematics, College of Science, Vietnam National University, Hanoi, Viet Nam; Department of Mathematics, National University of Singapore, 2 Science Drive 2, 117543 Singapore, Singapore

Abstract: Using variational methods we study the non-existence and multiplicity of non-negative solutions for a class of quasilinear elliptic equations of $p(x)$ -Laplacian type with nonlinear boundary conditions of the form $-\text{Div}(|u|^{p(x)-2}u) + |u|^{p(x)-2}u = 0$ in Ω , $|u|^{p(x)-2}u = g(x,u)$ on $\partial\Omega$ where Ω is a bounded domain with smooth boundary, n is the outer unit normal to $\partial\Omega$ and λ is a parameter. Furthermore, we want to emphasize that $g: \Omega \times \mathbb{R} \rightarrow [0, \infty)$ is a continuous function that may or may not satisfy the Ambrosetti-Rabinowitz-type condition. © 2010 Royal Society of Edinburgh.

Year: 2010

Source title: Proceedings of the Royal Society of Edinburgh Section A: Mathematics

Volume: 140

Issue: 2

Page : 259-272

Link: Scopus Link

Correspondence Address: Chung, N. T.; Department of Mathematics and Informatics, Quang Binh University, 312 Ly Thuong Kiet, Dong Hoi, Quang Binh, Viet Nam; email: ntchung82@yahoo.com

ISSN: 3082105

DOI: 10.1017/S030821050800070X

Language of Original Document: English

Abbreviated Source Title: Proceedings of the Royal Society of Edinburgh Section A: Mathematics

Document Type: Article

Source: Scopus

Authors with affiliations:

1. Chung, N.T., Department of Mathematics and Informatics, Quang Binh University, 312 Ly Thuong Kiet, Dong Hoi, Quang Binh, Viet Nam
2. Ng, Q.-A., Department of Mathematics, College of Science, Vietnam National University, Hanoi, Viet Nam, Department of Mathematics, National University of Singapore, 2 Science Drive 2, 117543 Singapore, Singapore

References:

1. Alves, C.O., Souto, M.A.S., Existence of solutions for a class of problems in $\mathbb{R}^N$ involving the $p(x)$ -Laplacian (2005) Contributions to Nonlinear Analysis, Progress in Nonlinear Differential Equations and Their Applications, 66, pp. 17-32. , Birkhäuser

2. Ambrosetti, A., Rabinowitz, P.H., Dual variational methods in critical points theory and applications (1973) *J. Funct. Analysis*, 4, pp. 349-381
3. De apoli, P., Mariani, M.C., Mountain pass solutions to equations of p-Laplacian type (2003) *Nonlin. Analysis*, 54, pp. 1205-1219
4. Ekeland, I., On the variational principle (1974) *J. Math. Analysis Applic.*, 47, pp. 324-353
5. Fan, X.L., Boundary trace embedding theorems for variable exponent Sobolev spaces (2008) *J. Math. Analysis Applic.*, 339, pp. 1395-1412
6. Fan, X.L., Han, X., Existence and multiplicity of solutions for $p(x)$ -Laplacian equations in $\mathbb{R}^N$ (2004) *Nonlin. Analysis*, 59, pp. 173-188
7. Fan, X.L., Zhang, Q.H., Existence of solutions for $p(x)$ -Laplacian Dirichlet problem (2003) *Nonlin. Analysis*, 52, pp. 1843-1852
8. Fan, X.L., Zhao, D., On the spaces $L_{p(x)}(y)$ and $W_{m,p(x)}(y)$ (2001) *J. Math. Analysis Applic.*, 263, pp. 424-446
9. Fan, X.L., Shen, J., Zhao, D., Sobolev embedding theorems for spaces $W_{k,p(x)}(y)$ (2001) *J. Math. Analysis Applic.*, 262, pp. 749-760
10. Fan, X.L., Zhang, Q.H., Zhao, D., Eigenvalues of $p(x)$ -Laplacian Dirichlet problem (2005) *J. Math. Analysis Applic.*, 302, pp. 306-317
11. Halsey, T.C., Electrorheological fluids (1992) *Science*, 258, pp. 761-766
12. Kováčik, O., Ńakosťk, J., On spaces $L_{p(x)}$ and $W_{1,p(x)}$ (1991) *Czech. Math. J.*, 41, pp. 592-618
13. Mihailescu, M., Existence and multiplicity of weak solutions for a class of denegerate nonlinear elliptic equations (2006) *Boundary Value Probl.*, 17, p. 41295
14. Mihailescu, M., Radulescu, V., A multiplicity result for a nonlinear degenerate problem arising in the theory of electrorheological fluids (2006) *Proc. R. Soc. Lond.*, A462, pp. 2625-2641
15. Musielak, J., Orlicz spaces and modular spaces (1983) *Lecture Notes in Mathematics*, 1034, Springer
16. Perera, K., Multiple positive solutions for a class of quasilinear elliptic boundary-value problems (2003) *Electron. J. Diff. Eqns*, 7, pp. 1-5
17. Ruzicka, M., (2002) *Electrorheological Fluids: Modeling and Mathematical Theory*, Springer
18. Struwe, M., (2000) *Variational Methods*, 2nd edn Springer
19. Zhang, Q.H., Existence of radial solutions for $p(x)$ -Laplacian equation in $\mathbb{R}^N$ (2006) *J. Math. Analysis Applic.*, 315, pp. 506-516