The hit problem for the modular invariants of linear groups

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Abstract: Let the mod 2 Steenrod algebra, A, and the general linear group, GL_k := GL(k, F_2), act on P_k := $F_2[x_1,...,x_k]$ with deg (x_i) = 1 in the usual manner. We prove that, for a family of some rather small subgroups G of GL_k, every element of positive degree in the invariant algebra P_k^G is hit by A in P_k. In other words, $(P_k^G)^+$?? A^+ ? P_k, where $(P_k^G)^+$ and A^+ denote respectively the submodules of P_k^G and ?a consisting of all elements of positive degree. This family contains most of the parabolic subgroups of GL_k. It should be noted that the smaller the group G is, the harder the problem turns out to be. Remarkably, when G is the smallest group of the family, the invariant algebra P_k^G is a polynomial algebra in k variables, whose degrees are ?? 8 and fixed while k increases. It has been shown by Hu'ng [Trans. Amer. Math. Soc. 349 (1997), 3893-3910] that, for G = GL_k, the inclusion $(P_k^{GL_k})^+$?? A^+ ? P_k is equivalent to a weak algebraic version of the long-standing conjecture stating that the only spherical classes in Q_0S^0 are the elements of Hopf invariant 1 and those of Kervaire invariant 1. ?? 2001 Elsevier Science.

Author Keywords: Dickson invariant; Invariant theory; M??i invariant; Steenrod algebra

Year: 2001

Source title: Journal of Algebra

Volume: 246

Issue: 1

Page : 367-384

Cited by: 3

Link: Scorpus Link

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ISSN: 218693

CODEN: JALGA

DOI: 10.1006/jabr.2001.8974

Language of Original Document: English

Abbreviated Source Title: Journal of Algebra

Document Type: Article

Source: Scopus

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