

# On a class of degenerate and singular elliptic systems in bounded domains

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**Abstract:** This paper deals with the nonexistence and multiplicity of nonnegative, nontrivial solutions to a class of degenerate and singular elliptic systems of the form  $\{(-\operatorname{div}(h_1(x) \nabla u) = \lambda F_u(x, u, v), \text{ in } \Omega; -\operatorname{div}(h_2(x) \nabla v) = \lambda F_v(x, u, v), \text{ in } \Omega)\}$  where  $\Omega$  is a bounded domain with smooth boundary  $\partial\Omega$  in  $\mathbb{R}^N$ ,  $N \geq 2$ , and  $h_i : \Omega \rightarrow [0, \infty)$ ,  $h_i \in L^1_{\text{loc}}(\Omega)$ ,  $h_i$  ( $i = 1, 2$ ) are allowed to have "essential" zeroes at some points in  $\Omega$ ,  $(F_u, F_v) = \lambda F$ , and  $\lambda$  is a positive parameter. Our proofs rely essentially on the critical point theory tools combined with a variant of the Caffarelli-Kohn-Nirenberg inequality in [P. Caldiroli, R. Musina, On a variational degenerate elliptic problem, NoDEA Nonlinear Differential Equations Appl. 7 (2000) 189-199]. © 2009 Elsevier Inc. All rights reserved.

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