

PATH INTEGRAL QUANTIZATION OF SELF INTERACTING SCALAR FIELD WITH HIGHER DERIVATIVES

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Abstract: *Scalar field systems containing higher derivatives are studied and quantized by Hamiltonian path integral formalism. A new point to previous quantization methods is that field functions and their time derivatives are considered as independent canonical variables. Consequently, generating functional, explicit expressions of propagators and Feynman diagrams in ϕ^3 theory are found.*

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1. INTRODUCTION

Field systems containing derivatives of order higher than first have more and more important roles with the advent of super-symmetry and string theories [1]. However, up to now path integral quantization method is almost restricted to fields with first derivatives [2, 3, 4].

The purpose of this paper is to apply the new ideal “*velocities have to be taken as independent canonical variables*” [5] to extending the method to self-interacting scalar field containing higher derivatives.

The paper is organized as follows: Section II presents the application of this quantization method to quantizing free scalar field. Section III is devoted to studying the Feynman diagrams of self-interacting scalar field. Section IV is for the drawn conclusion.

2. FREE SCALAR FIELD

Let us consider Lagrangian density for a free scalar field, containing second order derivatives

$$L = \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + \frac{1}{2\Lambda^2} \square \phi \square \phi, \quad (1)$$

where $\square$ is D’Alamber operator ($\square = \partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \Delta$), Δ is Laplacian and Λ is a parameter with dimension of mass. It will give a term with k^4 in the denominator of the corresponding Feynman propagator. This renders a finite result for some diagrams and, consequently, it may permit the introduction of convenient counter-terms to absorb the

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infinities which will appear when the limit Λ is taken.

The canonical momenta, conjugate to ϕ and $\dot{\phi}$, are respectively

$$\pi = \dot{\phi} - \frac{1}{\Lambda^2} \square \dot{\phi}; \quad s = \frac{1}{\Lambda^2} \square \phi. \quad (2)$$

Now, there are no constraints involved. We notice that $\dot{\phi}$ is now an independent canonical variable and then it has to be functionally integrated. Thus, the canonical Hamiltonian density becomes

$$\begin{aligned} \mathcal{H}_c &= \pi \dot{\phi} + s \ddot{\phi} - L \\ &= \pi X + \frac{1}{2} \Lambda^2 s^2 + s \nabla^2 \phi - \frac{1}{2} X^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2, \end{aligned} \quad (3)$$

where to avoid mistakes we have denoted the independent coordinate $\dot{\phi}$ by X .

The corresponding generating functional is given by

$$\begin{aligned} Z[J, K] &= N \int [d\phi] [ds] [d\pi] [dX] \exp \left\{ i \int d^4x \left[\pi \dot{\phi} + s \dot{X} - \pi X - \right. \right. \\ &\quad \left. \left. - s \nabla^2 \phi + \frac{1}{2} X^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} m^2 \phi^2 + J\phi + KX \right] \right\}. \end{aligned} \quad (4)$$

In this case, integrations over π and X are immediately calculated by using delta functionals and 4-dimensional Gaussian integral. Integration over ϕ is calculated by putting $\phi = \phi_c + \psi$, in which, ϕ_c is determined by field equation for extended Lagrangian, it means, satisfying

$$\left(m^2 + \square - \frac{1}{2\Lambda^2} \square \square \right) = J - \dot{K}. \quad (5)$$

The result is

$$\begin{aligned} Z[J, K] &= N_1 \exp \left\{ \frac{i}{2} \int d^4x \left[J(x) \frac{1}{\square + m^2 - \frac{1}{\Lambda^2} \square \square} J(x) \right. \right. \\ &\quad \left. \left. - K(x) \frac{\partial_0^2}{\square + m^2 - \frac{1}{\Lambda^2} \square \square} K(x) \right. \right. \\ &\quad \left. \left. + 2K(x) \frac{\partial_0}{\square + m^2 - \frac{1}{\Lambda^2} \square \square} J(x) \right] \right\}. \end{aligned} \quad (6)$$

The Feynman propagator $\langle 0 | T(\phi(x) \phi(x')) | 0 \rangle$ can be directly obtained by the usual expression

$$\begin{aligned} \langle 0 | T(\phi(x) \phi(x')) | 0 \rangle &= \frac{i^{-2}}{Z} \frac{\delta^2 Z}{\delta J(x) \delta J(x')} \Big|_{J, K=0} \\ &= - \frac{i}{m^2 + \square - \frac{1}{\Lambda^2} \square \square} \delta^4(x - x'). \end{aligned} \quad (7)$$

Since we have introduced a source for $\dot{\phi}$, it is also possible to calculate the following propagators

$$\langle 0 | T \left(\dot{\phi}(x) \dot{\phi}(x') \right) | 0 \rangle = \frac{i \partial_0^2}{m^2 + \square - \frac{1}{\Lambda^2} \square \square} \delta^4(x - x'), \quad (8)$$

$$\langle 0 | T \left(\phi(x) \dot{\phi}(x') \right) | 0 \rangle = \frac{-i \partial_0}{m^2 + \square - \frac{1}{\Lambda^2} \square \square} \delta^4(x - x'). \quad (9)$$

Propagator (7) is in agreement with the correct propagator by following the usual canonical procedure [6]. More over, when the limit Λ is taken, it has the usual form corresponding to the ordinary free scalar field (containing first derivatives) we have known before. The above propagators calculated explicitly is an important step to obtain Feynman diagrams and propagators of self-interacting scalar field in the next section.

3. SCALAR FIELD IN ϕ^3 THEORY

Now, we consider ϕ^3 self-interacting scalar field by adding an interacting term $L_{int} = -\frac{g}{6}\phi^3$ to the Lagrangian (1)

$$L = \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + \frac{1}{2\Lambda^2} \square \phi \square \phi + \frac{g}{6} \phi^3. \quad (10)$$

Since the interacting field L_{int} only depends on ϕ and the final form of the generating functional Z contains only field configuration $d\phi$ under the integrand, the generating functional $Z[J, K]$ with higher derivatives, in ϕ^3 interacting theory, is similar to the ones with first order derivatives. It means, the re-normalization generating functional [7] $Z[J, K]$ is

$$Z[J, K] = \frac{\exp \left[i \int L_{int} \left(\frac{1}{i} \frac{\delta}{\delta J} dx \right) \right] Z_0[J, K]}{\exp \left[i \int L_{int} \left(\frac{1}{i} \frac{\delta}{\delta J} dx \right) \right] Z_0[J, K] \big|_{J, K=0}}. \quad (11)$$

Since L_{int} also depends on ϕ , the formula of the S matrix still has form

$$S =: \exp \left[\int \phi_{int} K \frac{\delta}{\delta J(z)} \right] : Z[J, K] \big|_{J, K=0}, \quad (12)$$

where $K = \square + m^2 - \frac{1}{\Lambda^2} \square \square$.

So that, we can apply LSZ formula to the interaction between two in-particles and two out-particles. The scattering amplitude is

$$\begin{aligned} \langle f | S - 1 | i \rangle = & \int d^4 x_1 d^4 x_2 d^4 x'_1 d^4 x'_2 e^{i(k_1 x_1 + k_2 x_2 - k'_1 x'_1 - k'_2 x'_2)} K(x_1) K(x_2) \times \\ & \times K(x'_1) K(x'_2) \langle 0 | T(\phi(x_1) \phi(x_2) \phi(x'_1) \phi(x'_2)) | 0 \rangle_C, \end{aligned} \quad (13)$$

where $K(x_1) \tau(x_1, y) = -i \delta^4(x_1 - y)$.

Formula (13) is calculated explicitly through 4-point function (the procedure is the same as in [7])

$$\begin{aligned} \langle f | S - 1 | i \rangle = & (-ig)^2 \int d^4 y d^4 z \tau(y - z) \left[e^{i(k_1 y + k_2 y - k'_1 z - k'_2 z)} \right. \\ & + e^{i(k_1 y + k_2 z - k'_1 y - k'_2 z)} \\ & \left. + e^{i(k_1 y + k_2 z - k'_1 z - k'_2 y)} \right] + O(g^4), \end{aligned} \quad (14)$$

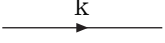
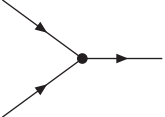
where

$$\tau(x-y) = \int \frac{d^4k}{(2\pi)} \frac{-i}{k^2 - m^2 + i\varepsilon + \frac{1}{\Lambda^2}k^4} e^{ik(x-y)}. \quad (15)$$

Substituting (15) into (14) and integrating over $dy dz$, we obtain

$$\begin{aligned} \langle f | S - 1 | i \rangle &= ig^2(2\pi)^4 \delta(k_1 + k_2 - k'_1 - k'_2) \\ &\times \left[\frac{1}{(k_1 + k_2)^2 - m^2 + \frac{1}{\Lambda^2}(k_1 + k_2)^4} \right. \\ &+ \frac{1}{(k_1 - k'_1)^2 - m^2 + \frac{1}{\Lambda^2}(k_1 - k'_1)^4} \\ &\left. + \frac{1}{(k_1 - k'_2)^2 - m^2 + \frac{1}{\Lambda^2}(k_1 - k'_2)^4} \right] + O(g^4). \end{aligned} \quad (16)$$

From (16), we have the following Feynman rules for the scattering amplitude

Diagrammatic representation	Factor in S matrix
Internal line	 $\frac{-i}{k^2 - m^2 + i\varepsilon + \frac{1}{\Lambda^2}k^4}$
External line	 1
Vertex	 $\frac{-i}{k^2 - m^2 + i\varepsilon + \frac{1}{\Lambda^2}k^4}$

In summary, by using above improved path integral quantization method, Feynman diagrams for self-interacting ϕ^3 scalar field are found. In general, when interacting term is more complicated, for example it contains derivatives of ϕ , Feynman diagrams will have two more new kinds of vertex, corresponding to interacting vertices $\dot{\phi} - \phi$ and $\dot{\phi} - \dot{\phi}$.

4. CONCLUSION

We have studied the Hamiltonian path integral formulation for scalar field with higher derivatives and also considered the system in ϕ^3 self-interaction. The new ideal is that time derivatives of field functions are considered as independent canonical variables. Generating functional and explicit expressions of propagators are calculated. Feynman diagrams for ϕ^3 interacting field are obtained explicitly. Extension of this result to electrodynamics (interacting with matter), string theory or gravity theory will be studied latter.

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