Acoustomagnetoelectric effect in a superlattice

Nguyen Quang Bau[1], Nguyen Van Hieu[1,*],
Nguyen Thi Thanh Huyen[1], Nguyen Dinh Nam[1], Tran Cong Phong[2]

[1]*Faculty of Physics, College of Science, VNU*
334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam
[2]*Hue University of Education, 32 Le Loi, Hue, Vietnam*

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Abstract. The acoustomagnetoelectric (AME) effect in a superlattice (SL) is investigated for an acoustic wave whose wavelength $\lambda = 2\pi/q$ is smaller than the mean free path l of the electrons and hypersound in the region $ql \gg 1$ (where q is the acoustic wave number) . The analytical expression for the AME current j^{AME} is calculated in the case of relaxation time of momentum τ is constant approximation. The result indicates that the existense of j^{AME} in a SL may be due to the finite gap band , and the periodicity of the electron spectrum along the SL axis. Numerical calculations have been done and the result is analysed for the GaAs/AlAs SL. All the results are compared with the normal bulk semiconductors (both theory and experiment) in the weak magnetic field region to show the difference.

1. Introduction

It is well known that, when an acoustic wave propagates through a conductor, it is accompanied by a transfer of energy and momentum to the conducting electrons. This gives rise to what is called the acoustoelectric effect. The study of acoustoelecric effect in the bulk semiconductor has received a lot of attention[1-4]. Recently, Mensah has investigated this effect in a superlattice [5] and there has been a growing interest in observing this effect in mesoscopic structures [6-8]. However, in the presence of the magnetic field the acoustic wave propagating in the conductor can produce another effect called the acoustomagnetoelectric (AME) effect. The AME effect is creating an AME current (if the sample is short circuited in the Hall direction), or an AME field (if the sample is open) when a sample placed in a magnetic field $\vec{H}$ carries an acoustic wave propagating in a direction perpendicular to $\vec{H}$.

The AME effect was first foreseed theoretically by Grinberg and Kramer [9] for bipolar semiconductors and was observed experimentally in bismuth by Yamada [10]. In past times, there are more and more interests in studying and discovering this effect, such as in a monopolar semiconductor [11], and in a Kane semiconductor [12] in this specimen they observed that the AME effect occurs mainly because of the dependence of the electron relaxation time τ on the energy and when $\tau = \text{constant}$,

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the effect vanishes. Like the classical magnetic field, the effect also exists in the case of a quantized magnetic field. Recently, D Margulis and A Margulis [13,14] have studied the quantum acoustomagnetoelectric (QAME) effect due to Rayleigh sound waves.

The AME effect is similar to the Hall effect in the bulk semiconductor, where, the sound flux $\vec{\Phi}$ plays the electric current $\vec{j}$ role. The essence of the AME effect is due to the existense of partial current generated by the different energy groups of electrons, when the total acoustoelectric (longitudinal) current in specimen is equal to zero. When this happens, the energy dependence of the electron momentum relaxition time will cause average mobilities of the electrons in the partial current, in general, to differ, if an external magnetic field is perpendicular to the direction of the sound flux, the Hall currents generated by these groups will not, in general, compensate one another, and a non-zero AME effect will result. The AME effect problem in bulk semiconductors for the case $ql \gg 1$ (where q is the acoustic wave numbe, l is the mean fee path) has been investigated [11,12,15]. The AME effect in a superlattice still, however, opens for studying, in this paper, we examine this effect in a superlattice for the case of electron relaxation time is not dependent on the energy. Futhermore, we think that the research of this effect may help us to understand the properties of SL material. It will be seen that, due to the anisotropic nature of the dispersion law, the AME effect is obtained at τ = constant. It is also nonlinear dependent on the SL parameters. Numerical calculations are carried out a specific GaAs/AlAs SL to clarify our results.

The paper is organized as follows. In section 2 we outline the theory and conditions necessary to solve the problem. In section 3 we discuss the results and in section 4 we have some conclusion.

2. Acoustomagnetoelectric current

Following the method developed in [6] we calculate the AME current in a SL. The acoustic wave will be considered as a hypersound in the region $ql \gg 1$ and then treated as monochromatic phonons (frequency ω_q). The problem will be solved in the quasi-classcal case, i.e. $2\Delta \gg \tau^{-1}$ (2Δ is the width of the miniband, τ is the electron relaxation time). The magnetic field will also be considered classically, i.e. $\Omega < \nu$, $\hbar\Omega \ll k_B T$ (ν is the frequency of electron collisions, Ω is the cyclotron frequency), and weak, thus limiting ourselves to the linear approximation of $\vec{H}$.

The density of the acoustoelectric current in the presence of magnetic field can be written in the form [15]

$$j^{AE} = \frac{2e}{(2\pi)^3} \int U^{AE} \psi_i d^3\vec{p}, \tag{1}$$

where

$$U^{AE} = \frac{2\pi\Phi}{\omega_q v_s}\Big\{ \mid G_{\vec{p}-\vec{q},\vec{p}} \mid^2 [f(\epsilon_{n,\vec{p}-\vec{q}}) - f(\epsilon_{n,\vec{p}})]\delta(\epsilon_{n,\vec{p}-\vec{q}} - \epsilon_{n,\vec{p}} + \omega_{\vec{q}})$$

$$+ \mid G_{\vec{p}+\vec{q},\vec{p}} \mid^2 [f(\epsilon_{n,\vec{p}+\vec{q}}) - f(\epsilon_{n,\vec{p}})]\delta(\epsilon_{n,\vec{p}+\vec{q}} - \epsilon_{n,\vec{p}} - \omega_{\vec{q}})\Big\}. \tag{2}$$

Here Φ is the sound flux, v_s is the velocity sound, $f(\epsilon_{n,\vec{p}})$, $\epsilon_{n,\vec{p}}$ are the distribution function and the energy of the electron, respectively, n denotes quantization of the energy spectrum, $G_{\vec{p}\pm\vec{q},\vec{p}}$ is the matrix element of the electron-phonon interaction and ψ_i is the root of the kinetic equation given by[8]

$$\frac{e}{c}(\vec{V} \times \vec{H})\frac{\partial \psi_i}{\partial p} + \widehat{W_{\vec{p}}}\{\psi_i\} = \vec{V}_i. \tag{3}$$

Here $\vec{V_i}$ is the electron velocity and $\widehat{W_{\vec{p}}}\{...\} = (\partial f/\partial\epsilon)^{-1}\widehat{W_{\vec{p}}}(\partial f/\partial\epsilon...)$. The operator $\widehat{W_{\vec{p}}}$ is the collision operator describing relexation of the non-equilibrium distribution of electron, and $\widehat{W_{\vec{p}}}$ is assumed to be Hermitian [16]. In the 'τ-approximation constant', the collision operator has form $\widehat{W_{\vec{p}}} = 1/\tau$. We shall seek the solution of Eq.(3) as

$$\psi_i = \psi_i^{(0)} + \psi_i^{(1)} + ...$$ (4)

substituting Eq.(4) into Eq.(3) and solving by the method of iteration, we get for the zero and the first approximation. Inserting into Eq.(1) and taking into account the fact that $\mid G_{\vec{p},\vec{p'}}\mid^2 = \mid G_{\vec{p'},\vec{p}}\mid^2$, we obtain for the acoustoelectric current the expression

$$j_i^{AE} = -\frac{e\Phi}{2\pi^2 v_s \omega_{\vec{q}}} \int \mid G_{\vec{p}+\vec{q},\vec{p}}\mid^2 [f(\epsilon_{n,\vec{p}+\vec{q}}) - f(\epsilon_{n,\vec{p}})]\times$$

$$\times [V_i(\vec{p}+\vec{q})\tau - V_i(\vec{p})\tau]\delta(\epsilon_{n,\vec{p}+\vec{q}} - \epsilon_{n,\vec{p}} - \omega_{\vec{q}})d^3\vec{p}$$

$$-\frac{e^2\Phi\tau^2}{2\pi^2 mc\omega_{\vec{q}}v_s} \int \mid G_{\vec{p}+\vec{q},\vec{p}}\mid^2 [f(\epsilon_{n,\vec{p}+\vec{q}}) - f(\epsilon_{n,\vec{p}})]\times$$

$$\times [(\vec{V}(\vec{p}+\vec{q})\times\vec{H})_i - (\vec{V}(\vec{p})\times\vec{H})_i]\delta(\epsilon_{n,\vec{p}+\vec{q}} - \epsilon_{n,\vec{p}} - \omega_{\vec{q}})d^3\vec{p}.$$ (5)

The matrix element of the electron-phonon interaction for $qd \ll 1$ (d is the period of the SL) is given as $\mid G_{\vec{p}+\vec{q},\vec{p}}\mid^2 = \frac{\Lambda^2\vec{q}^2}{2\sigma\omega_{\vec{q}}}$, where Λ is the deformation potential constant and σ is the density of the SL.

In solving Eq.(5) we shall consider a situation whereby the sound is propagating along the SL axis (oz), the magnetic field $\vec{H}$ is parallel to the (ox) axis and the AME current appears parallel to the (oy) axis. Under such orientation the first term in Eq.(5) is responsible for the acoustoelectric current and solution is found in [6]. The second term is the AME current and is expressed as

$$j_y^{AME} = -\frac{e\Phi\vec{q}^2\tau^2\Lambda^2\Omega}{4\pi v_s\omega_{\vec{q}}^2\sigma} \int [f(\epsilon_{n,\vec{p}+\vec{q}}) - f(\epsilon_{n,\vec{p}})]\times$$

$$\times [V_z(\vec{p}+\vec{q}) - V_z(\vec{p})]\delta(\epsilon_{n,\vec{p}+\vec{q}} - \epsilon_{n,\vec{p}} - \omega_{\vec{q}})d^3\vec{p},$$ (6)

where $\Omega = eH/mc$.

The distribution function $f(\epsilon_{n,\vec{p}})$ of degenerate electrons gas is given by

$$f(\vec{p}) = \Theta(\epsilon_F - \epsilon_{n,\vec{p}}) = \begin{cases} 0 & \epsilon_{n,\vec{p}} > \epsilon_F \\ 1 & \epsilon_{n,\vec{p}} < \epsilon_F \end{cases}$$ (7)

where ϵ_F is Fecmi energy. The energy spectrum $\epsilon_{n,\vec{p}}$ of electron in the SL is given using the usual notation by [17]

$$\epsilon_{n,\vec{p}} = \frac{\vec{p}_\perp^2}{2m} + \Delta_n(1 - \cos(p_z d)).$$ (8)

Here, $\vec{p}_\perp$ and p_z are the transverse and longitudinal (relative to the SL axis) components of the quasi-momentum, respectively; Δ_n is the half width of the nth allowed miniband, m is the effective mass of electron.

We assume that electrons are confined to the lowest conduction miniband ($n = 1$) and omit the miniband indicates. This is to say that the field does not induce transitions between the filled and

empty minibands, thus the Δ_n can be written the Δ.

Substituting Eq.(7) and (8) into Eq.(6) we obtain for the AME current satisfying the condition

$$\epsilon_F > \Delta \left[1 - \frac{\omega_{\vec{q}}}{2\Delta} + \left(\cos\frac{qd}{2}(1 - \frac{\omega_{\vec{q}}}{2\Delta\sin(\frac{qd}{2})})^2 \right)^{1/2} \right] + \omega_{\vec{q}} + \frac{p_\perp^2}{2m}. \qquad (9)$$

The inequality (9) is condition for acoustic wave $\vec{q}$ to the AME effect exists. Therefore, we have obtained the expression of the AME curent

$$j_y^{AME} = \frac{e\Phi \, \Lambda^2 \, \vec{q}^2 \tau^2 m\Omega}{2\pi v_s \omega_{\vec{q}} \sigma} \left[1 - (\frac{\omega_{\vec{q}}}{2\Delta\sin(\frac{qd}{2})})^2 \right]^{1/2}, \qquad (10)$$

the Eq.(10) is the AME current in SL for the case degenerate electron gas, that is only obtained if the condition of inequality (9) is satisfied. We can see that the dependence of the AME current on the frequency $\omega_{\vec{q}}$ is nonlinear.

3. Numerical results and discussions

The parameters used in the calculations are as follow [12,13,16]: $\Delta = 0.1eV, d = 100 \, A^o, \tau = 10^{-12}s, m = 0.067m_0$, m_0 being the mass of free electron, $H = 2.10^3 \, A \, m^{-1}$, $\Phi = 10^4 \, W \, m^{-2}, v_s = 5370 \, m \, s^{-1}$.

The result in Eq.(10) can be written in terms of the acoustoelectric current

$$j_y^{AME} = j_z^{AE}\Omega\tau, \qquad (11)$$

the AME current depends on the magnetic field $\vec{H}$, the quantity $\Omega\tau$ serving as a measure of the magnetic strength. The ratio of j^{AME}/j^{AE} is equal to $\Omega\tau$. This result is quite interesting as a similar ratio calculated for the case of the QAME due to the Rayleigh sound wave was of that order [15]. In their case, $\Omega\tau \gg 1$ (quantized magnetic field) and the sample was a bulk material, bulk semiconductor [12,13].
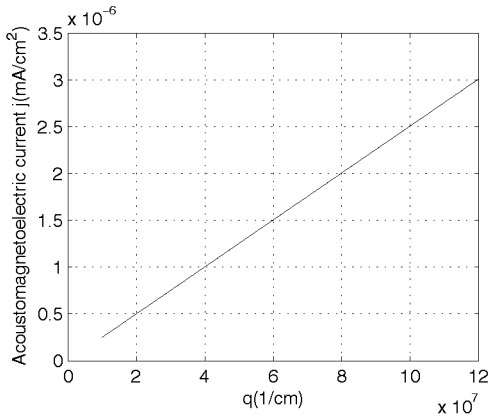
Fig. 1. The dependence of AME current on the q for the case of the bulk semiconductor.
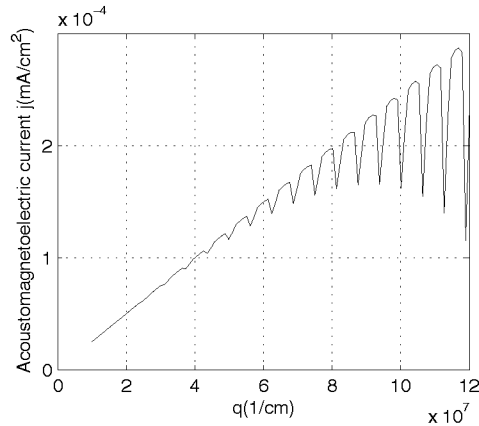
Fig. 2. The dependence of AME current on the q for the case of the superlattice.

It is plausible that mechanism responsible for the existence of the AME effect in a SL may be due to the finite band gap and the periodicity of the electron spectrum along the z axis and not the

dependence of τ on $\epsilon_{\vec{p}}$ [12,15]. The calculation was done on the basis of τ=constant and according to [11] the AME effect should be zero. However, for $\omega_{\vec{q}} > \Delta$ when the SL behaves as a bulk monopolar semiconductor with the parabolic law of dispersion, $j^{AME} \to 0$ as expected for τ=constant [11]. This is readily deduced from the conservation laws. The non-linear dependence of j^{AME} on the SL parameters Δ and d and the frequency $\omega_{\vec{q}}$ and particularly the strong spatial dispersion of j^{AME} once again can only be attributed to the finite band gap and periodicity of the energy spectrum of electron along the z axis.

Figure 1 shows the dependence of the AME current on the acoustic wave number in the case of the bulk semiconductor n-InSb[14]. It can be see from figure 1, When the $\vec{q}$ rises up, the AME current increases linear and value of AME current is very small, approximation 10^{-6} mA cm^{-2}. However, in figure 2 when we have investigated for the case of superlattice, there appear distinct maxima and the value of the AME current is larger than that of the AME current in the case of bulk semiconductor n-InSb. The cause of the difference between the bulk semiconductor and the superlattice, because of the low-dimensional systems characteristic, meanly, in the low-dimensional systems the energy spectrum of electron is quantized, like this, the AME effect have been appeared in SL for the case degenerates electrons gas, and note that it exists even if the relaxation time τ of the carrier independs on the carrier energy and has a strong spatial dispersion. In the limit case at $\omega_q = 10^{13}$ s^{-1}, and $H = 2.10^3$ Am^{-1} the AME current is obtained the value about 10^{-4} mA cm^{-2}, this value fits with the experimental result in [13].

4. Conclusion

In this paper, we have obtained analytical expressions for the AME current in a SL for the case of the degenerate electron gas. The strong dependences of j_y^{AME} on the frequency Ω of the magnetic field, $\omega_{\vec{q}}$ of the acoustic wave, the SL parameters Δ and d are the miniband half width and the period of the SL, respectively. The result shows that it exists even if the relaxation time τ of the carrier does not depend on the carrier energy and has a strong spatial dispersion, which result is different compared to those obtained in bulk semiconductor [11,12], according to [11] in the case $\tau = constant$ the effect only exists if the electron gas is non-degenerate, if the electron gas is degenarate, the effect does not appear. However, our result indicates that in (SL) the AME effect exists both the non-degenerate and the degenerate electron gas when the relaxation time τ of the carrier does not depend on the carrier energy. In addition, our analysis shows that the result has value, which is smaller than its in [5,12,15,17] and increases linearly Ω of the magnetic field. This result is similar to the semiconductor and the superlattice for the case of the non-degenerate electron gas in the weak magnetic field region [12,14,15]. Unlike the semiconductor, in the SL the AME current is non-linear with the acoustic wave $\vec{q}$. Especially, in the limit case at $\omega_q = 10^{13}$ s^{-1}, and $H = 2.10^3$ Am^{-1} the AME current is obtained the value about 10^{-4} mA cm^{-2}, this value fits with the experimental result in [13].

The numerical result obtained for a GaAs/AlAs SL shows that the AME effect exists when the $\vec{q}$ of the acoustic wave complies with specific conditions (9) which condition dependens on the frequency of the acoustic wave $\omega_{\vec{q}}$, Fermi energy, the mass of electrons, the miniband half width Δ and the period of the SL d. Namely, to have AME current , the acoustic phonons energy are high enough and satisfied in the some interval to impact much momentum to the conduction electrons.

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