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Finite volume method for long wave runup: 1D model

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Abstract. A numerical model using the 1D shallow water equations was developed for the simulation of long wave propagation and runup. The developed model is based on the Finite Volume Method (FVM) with an application of Godunov - type scheme of second order of accuracy. The model uses the HLL approximate Riemann solver for the determination of numerical fluxes at cell interfaces. The model was applied to the simulation of long wave propagation and runup on a plane beach and simulated results were compared with the published experimental data. The comparison shows that the present model has a power of simulation of long wave propagation and runup on beaches.

Keywords: Finite Volume Method; Shallow Water Model; Wave Runup.

1. Introduction

Long wave runup on beaches is one of the hot challenging topics recently, for the ocean and coastal engineering researchers. Frequently, engineers face to problems related to the simulation or determination of wave runup in general, and long wave runup in particular for practical purposes, such as design of sea wall, coastal structures, etc. Therefore, development of a good model capable of simulation of wave runup is worth for practical usage as well as for indoor researches.

Researchers have developed various analytical and numerical models based on the depth integrated shallow water equations to explain the physical processes. Notable analytical results include the one-dimensional solution of Carrier and Greenspan (1958) for periodic wave reflection from a plane beach [1] and the asymmetric solution by Thacker (1981) [6] for wave resonance in a circular parabolic basin. Synolakis (1987) [5] provided valuable experimental data of long wave runup on a plane beach, which then were well known among coastal engineering community, who do the job related with numerical modeling of coastal hydrodynamic processes. Analytical approach provides exact solution for idealized situation of geometry and offers insights into the physical processes. Numerical models provide approximate solutions in more general settings suitable for practical applications [9]. However, the main challenge lies in the treatment of the moving waterline and flow discontinuity when the water climbs up and down on beaches.

So far, many researchers have developed models for the simulation of wave runup.

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Shuto and Goto (1978) [4] used finite difference method with a staggered scheme and a Lagrangian description of the moving shoreline; Liu et al. (1995) modeled runup through flooding and drying of the cells in response to adjacent water level changes [3]. Titov and Synolakis (1995, 1998) [7, 8] proposed VTCS-2 model using the splitting technique and characteristic line method. Hu et al. (2000) [2] developed an 1D model using FVM with a Godunov-type upwind scheme to simulate the wave overtopping of seawall. Wei el al. (2006) presented a model for long wave runup using exact Riemann solver [9].

In this study, a numerical model is developed using FVM and the robust approximate Riemann solver HLL (Harten, Lax and van Leer) for the simulation of long wave runup on a beach. The model is verified for the case of experiment proposed by Synolakis (1987). Comparisons are carried out between simulated results and experimental data (Synolakis, 1987) [5]. The details of this study are given below.

2. Numerical model

2.1. Governing equation

The present study considers One-dimensional (1D) depth-integrated Shallow water equations in the Cartesian coordinate system (x,t). The conservation form of the 1D non-linear shallow water equations is written as

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}=\mathbf{S} \tag{1}$$

where $\mathbf{U}$ is the vector of conserved variables; $\mathbf{F}$ is the flux vectors; and $\mathbf{S}$ is the source term. The explicit form of these vectors is explained as follows:

$$\mathbf{U}=\begin{bmatrix} H \\ Hu \end{bmatrix}, \mathbf{F}=\begin{bmatrix} Hu \\ Hu^2+\frac{1}{2}gH^2 \end{bmatrix}, \mathbf{S}=\begin{bmatrix} 0 \\ gH\dfrac{\partial h}{\partial x}-\dfrac{\tau_x}{\rho} \end{bmatrix} \tag{2}$$

where g : gravitational acceleration; ρ : water density; h: still water depth; H: total water depth, $H=h+\eta$ in which $\eta(x,t)$ is the displacement of water surface from the still water level; τ_x : bottom shear stress given by:

$$\tau_x=\rho C_f u|u|, C_f=\frac{gn^2}{H^{1/3}} \tag{3}$$

where n: Manning coefficient for the bed roughness.

2.2. Numerical scheme

The finite volume formulation imposes conservation laws in a control volume. Integration of Eq. (1) over a cell with the application of the Green's theorem, gives:

$$\int_\Omega \frac{\partial \mathbf{U}}{\partial t}d\Omega+\int_\Gamma \mathbf{F}\cdot\mathbf{n}d\Gamma=\int_\Omega \mathbf{S}d\Omega \tag{4}$$

where Ω : cell domain; Γ : boundary of Ω ; $\mathbf{n}$: normal outward vector of the boundary.

Taking the time integration of Eq. (4) over duration Δt from t_1 to t_2, we have:

$$\begin{aligned}&\int_\Omega \mathbf{U}(x,t_2)d\Omega-\int_\Omega \mathbf{U}(x,t_1)d\Omega\\&+\int_{t_1}^{t_2}dt\int_\Gamma \mathbf{F}\cdot\mathbf{n}d\Gamma=\int_{t_1}^{t_2}dt\int_\Omega \mathbf{S}d\Omega\end{aligned} \tag{5}$$

Considering the case of one-dimensional model with cell size of Δx, from Eq. (5) we can deduce:

$$\begin{aligned}&\frac{1}{\Delta x\Delta t}\int_{x_i-\frac{\Delta x}{2}}^{x_i+\frac{\Delta x}{2}}\mathbf{U}(x,t_2)dx-\frac{1}{\Delta x\Delta t}\int_{x_i-\frac{\Delta x}{2}}^{x_i+\frac{\Delta x}{2}}\mathbf{U}(x,t_1)dx\\&+\frac{1}{\Delta x\Delta t}\int_{t_1}^{t_2}\left[\mathbf{F}\left(x+\tfrac{\Delta x}{2},t\right)-\mathbf{F}\left(x-\tfrac{\Delta x}{2},t\right)\right]dt\\&=\frac{1}{\Delta x\Delta t}\int_{t_1}^{t_2}dt\int_{x_i-\frac{\Delta x}{2}}^{x_i+\frac{\Delta x}{2}}\mathbf{S}dx\end{aligned} \tag{6}$$

Note that the integral $\frac{1}{\Delta x \Delta t}\int_{x_i-\frac{\Delta x}{2}}^{x_i+\frac{\Delta x}{2}} \mathbf{U}(x,t_2)dx$ is exactly the cell averaged value of $\mathbf{U}$ at time t_2, divided by Δt. The present model uses uniform cells with dimension Δx, thus, the integrated governing equations (6) with a time step Δt can be approximated with a half time step average for the interface fluxes and source term to become $\frac{1}{2}$:

$$\mathbf{U}_i^{k+1} = \mathbf{U}_i^k - \frac{\Delta t}{\Delta x}\left[\mathbf{F}_{i+1/2}^{k+1/2} - \mathbf{F}_{i-1/2}^{k+1/2}\right] + \Delta t \mathbf{S}_i^{t+1/2} \quad (7)$$

where i is index at the cell center; k denotes the current time step; the half indices $i+1/2$ and $i-1/2$ indicate the cell interfaces; and $k+1/2$ denotes the average within a time step between k and $k+1$. Note that, in Eq. (7) the variables $\mathbf{U}$ and source term $\mathbf{S}$ are cell-averaged values (we use this meaning from now on).

To solve the equation (7), we need to estimate the numerical fluxes $\mathbf{F}_{i+1/2}^{k+1/2}$ and $\mathbf{F}_{i-1/2}^{k+1/2}$ at the interfaces. In this study, we use the Godunov-type scheme for this purpose. According to the Godunov-type scheme, the numerical fluxes at a cell interface could be obtained by solving a local Riemann problem at the interface. The Godunov scheme can be expressed as:

$$\mathbf{F}_{i+1/2} = \mathbf{F}\left(\mathbf{U}_{i+1/2}^L, \mathbf{U}_{i+1/2}^R\right)_{x/t=0} \quad (8)$$

where $\mathbf{F}(\ )$ represents the numerical flux at the cell interface obtained by solving a local Riemann problem using the data $\mathbf{U}_{i+1/2}^L$ and $\mathbf{U}_{i+1/2}^R$ on each side of the cell interface. There are a number of approximate Riemann solvers proposed by different authors, such as Osher, Roe, etc. In this study, we use the HLL approximate Riemann solver. The formula for the solver is given as:

$$\mathbf{F}^* = \frac{s_R \mathbf{F}_L - s_L \mathbf{F}_R + s_L s_R(\mathbf{U}^R - \mathbf{U}^L)}{s_R - s_L} \quad (9)$$

$$s_L = \min\left\{u_L - C_L, u^* - C^*\right\} \quad (10)$$

$$s_R = \max\left\{u_R + C_R, u^* + C^*\right\} \quad (11)$$

$$u^* = \frac{u_L + u_R}{2} + (C_L - C_R) \quad (12)$$

$$C^* = \frac{C_L + C_R}{2} - \frac{u_R - u_L}{4} \quad (13)$$

where $\mathbf{F}^*$ denotes the HLL approximate Riemann solver; u_L and u_R are respectively the depth averaged velocities of water flow at left and right side of the cell interface; C_L and C_R are the shallow water wave speeds at left and right side of the interface.

In this study, we used three regions of wave speed to estimate the cell interface fluxes as follows:

$$\mathbf{F}_{i+1/2} = \begin{cases} \mathbf{F}_L & s_L \geq 0 \\ \mathbf{F}^* \quad \text{if} & s_L < 0 < s_R \\ \mathbf{F}_R & s_R \leq 0 \end{cases} \quad (14)$$

To get a second order of accuracy for the numerical model, $\mathbf{U}_{i+1/2}^L$ and $\mathbf{U}_{i+1/2}^R$, u_L and u_R, C_L and C_R are interpolated by using a linear reconstruction method based on the averaged values at cell centers with the usage of the TVD-type limiter, which is the average of Min-mode limiter and Roe limiter. For the wet and dry cell treatment, we use a minimum wet depth, the cell is assumed to be dry when its water depth less than the minimum wet depth (in this study we choose minimum wet depth of 10^{-5}m).

3. Simulation results and discussion

3.1. Experimental condition

A numerical experiment is carried out for the condition similar to the experiment done by Synolakis (1987). In this experiment, there was a beach having a slope of 1:19.85 connected to a horizontal bottom with water depth of $h = 1$m. The toe of the beach located at distance $x_2 / h = 19.85$ and shoreline was at $x = 0$. A solitary wave with the height of $A / h = 0.3$ was generated at $x_1 / h = 24.42$ coming to the beach from the part of constant water depth. The experiment provided with experimental data of water surface profile at different time. Fig. 1 shows the sketch of the experiment.

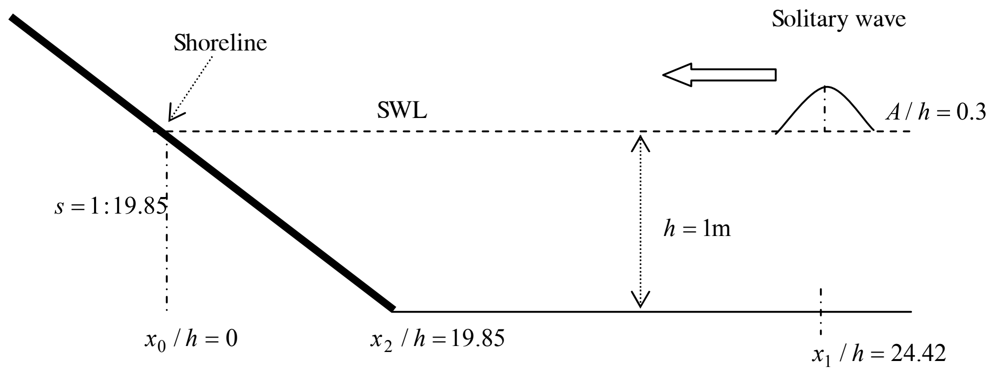

Fig. 1. Sketch of Synolakis's experiment.

For the numerical simulation, the initial solitary wave is simulated by the solitary wave formula as:

$$\eta(x,0) = \frac{A}{h}\operatorname{sech}\left[\sqrt{\frac{3A}{4h^3}}(x - x_1)\right] \qquad (15)$$

$$u(x,0) = \eta(x,0)\sqrt{\frac{g}{h}} \qquad (16)$$

The computation domain is discretized into cells in a regular mesh with space step $\Delta x = 0.1$m and the simulation is carried out with the initial condition given by equations (15) and (16). Simulated results of water surface profile are recorded for comparing with the experimental data.

3.2. Results and discussion

Fig. 2 shows the initial free surface simulated by the numerical model.

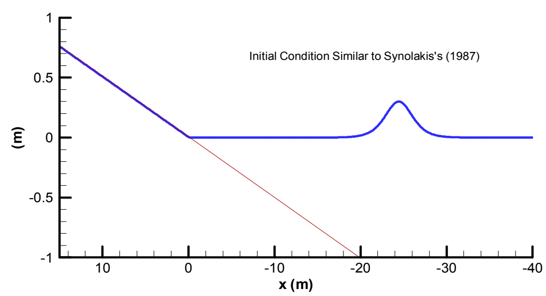

Fig. 2. Initial free surface of the simulation.

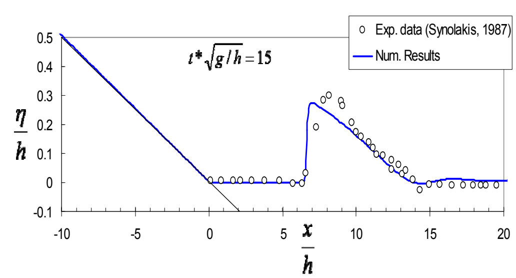

Fig. 3. Comparison with experimental data: near breaking location.

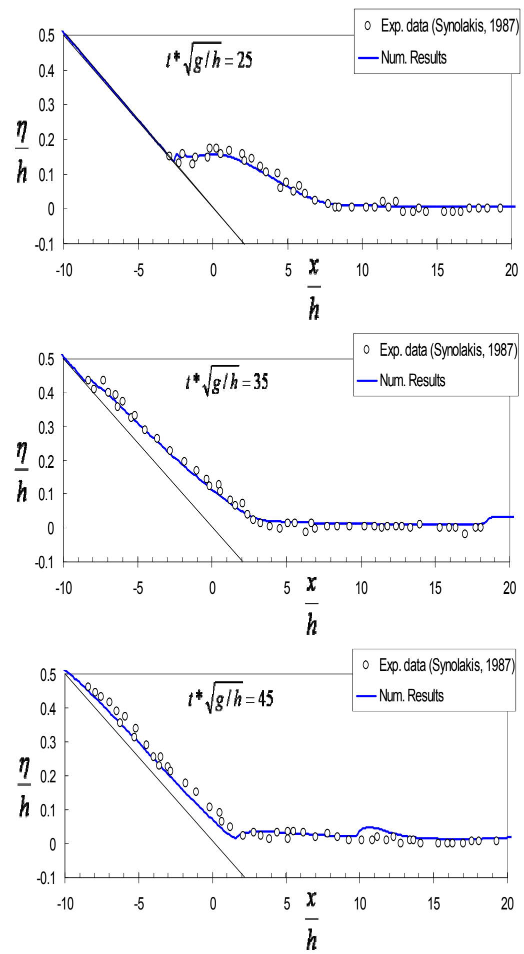

Fig. 4. Comparison with experimental data: runup phase.

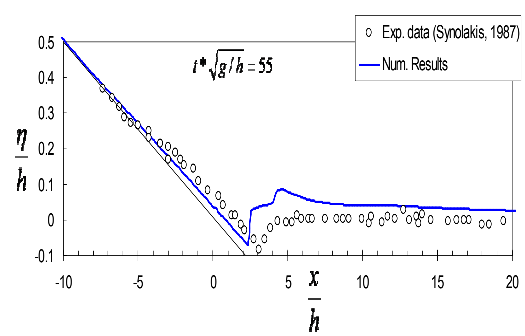

Fig. 5. Comparison with experimental data: rundown phase.

Fig. 3 shows the comparison between simulated results and experimental data of free surface profile near the breaking location. It is seen that simulated results have some discrepancy at the wave crest compared to the experimental data. This could be due to the limitation of the shallow water equation itself in simulation of wave dispersion and breaking. After that, in side the surf zone, computed results agree very well with the experimental data, especially during the runup process on the beach (see Fig. 4 at normalized time 25, 35, 45). The highest runup attains at normalized time of 45 and the highest runup is of 0.5m. This result is about 1.6 times of the initial wave height. The agreement between simulated results and experimental data during the time of runup process could be explained as due to conservation of mass and momentum ensured in the present model using the conserved FVM.

For the simulation of long wave runup on beaches, in practice, the most important thing is correctly simulated runup process and the highest climb up of water front. Although simulating the wave profile in the breaking zone is not well, the present model is still capable of simulation of wave runup process on the beach, specially the highest runup could be well simulated by the model. This is one of the practical purposes.

At the stage of rundown (see Fig. 5 at the normalized time of 55), the water including the position of shoreline and inundation depth on the beach is still well simulated. Thus, the developed model with the FVM proposed in this study has a power of expansion to a two-dimensional model and is also capable of simulation of non-linear wave runup, rundown processes including the prediction of highest runup of water.

4. Conclusions

A FVM based numerical model has been successfully developed for the simulation of long wave propagation and runup. This model specially well simulates the highest runup of water and inundation depth on the beach during runup and rundown processes.

The good agreement between the simulated results and experimental data reveals that the model has a potential for practical uses and should be studied further in order to expand to a two-dimensional model for various purposes in practice, such as simulation of Tsunami runup and inundation on coastal areas, flooding due to storm surge, etc.

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