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# Principles of Quantum Mechanics

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SECOND EDITION

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ĐẠI HỌC QUỐC GIA HÀ NỘI  
TRUNG TÂM THÔNG TIN THƯ VIỆN

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