NON-LINEAR ANALYSIS OF MULTILAYERED REINFORCED COMPOSITE PLATES

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Abstract. This paper deals with the analysis of non-linaer multilayered reinforced composite plates with simply supported along its four edges by Bubnov - Galerkin and Finite Element Methods. Numerical results are presented for illustrating theoretical analysis of reinforced and unreinforced laminated composite plates.

Keywords: Stiffened laminated composite plate, multilayered reinforced composite plates

1. Introduction

Multilayered reinforced composite plates are used extensively in Naval, Aerospace, Automobile applications and in Civil engineering.v.v... Today, analysis of linear laminated composite plates has been studied by many authors. However, the analysis of non-linear laminated composite plates has received comparatively little attention [3, 4, 5, 6,...] specially for analysis of non-linear stiffened laminated composite plates and shells subjected to distributed transverse loads. This problem is studied in the present paper.

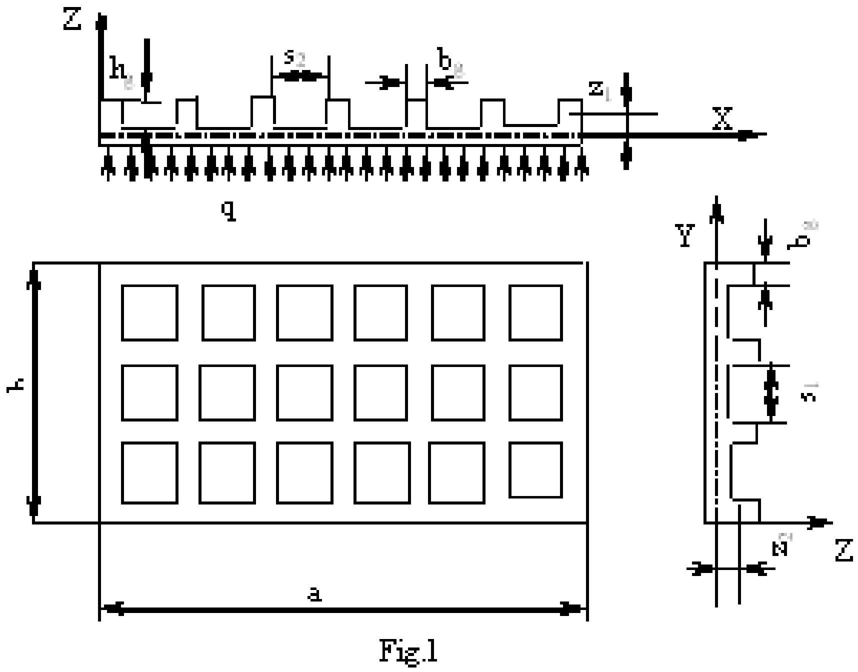

Fig.1

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2. Governing equations of laminated plates

Let's consider a rectangular multilayered reinforced composite plate, in which each layer is made of unidirectional composite material and stiffeners are made by composite material. This plate is subjected to distributed transverse loads (Figure 1).

For multilayered reinforced composite plates working in the elastic state the relation between internal force and deformation is of the form

$$\{\overline{\sigma}\} = \left[\overline{D}\right]\{\varepsilon\} \tag{1}$$

where

$$\{\overline{\sigma}\} = \left\{N_x \ N_y \ N_{xy} \ M_x \ M_y \ M_{xy}\right\}^T$$

$\left[\overline{D}\right]$ - Matrix of stiffness constants of multilayered reinforced composite plates

$$\left[\overline{D}\right] = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \tag{2}$$

in which

$$(A_{ij}, B_{ij}, D_{ij} = \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} (Q_{ij})_k (1, z, z^2) dz \quad (i, j = 1, 2, 6),$$

$\{\varepsilon\}$ - the deformation of point of the middle surface.

The strain - displacement relations in the non-linear theory are of the form

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\Big(\frac{\partial w}{\partial x}\Big)^2, \quad \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\Big(\frac{\partial w}{\partial y}\Big)^2, \quad \gamma_{xy} = \Big(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\Big) + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \tag{3}$$

$$k_x = -\frac{\partial^2 w}{\partial x^2}, \quad k_y = -\frac{\partial^2 w}{\partial y^2}, \quad k_{xy} = -2\frac{\partial^2 w}{\partial x \partial y},$$

where u, v, w are the middle displacements along the x, y and z axis respectively.

For a plate simply supported on all edges, the following boundary condition are in posed

+ At edges $x = 0$, $x = a$: $\quad w = 0; \quad v = 0; \quad M_x = 0;$ (4)
+ At edges $y = 0$, $y = b$: $\quad w = 0; \quad u = 0; \quad M_y = 0;$ (5)

3. Bubnov - Galerkin methods

According to Lekhnistki theory when expanding internal forces - deformations (1), we obtain the expression for stress resultants and flexion moments of multilayered reinforced composite plates

$$\begin{aligned} N_x &= (A_{11} + E_1 A_1/s_1)\varepsilon_x + A_{12}\varepsilon_y + (E_1 A_1/s_1) z_1 k_x, \\ N_y &= (A_{22} + E_2 A_2/s_2)\varepsilon_y + A_{12}\varepsilon_x + (E_2 A_2/s_2) z_2 k_y, \\ N_{xy} &= A_{66}\gamma_{xy}, \\ M_x &= (D_{11} + E_1 I_1/s_1) k_x + D_{12} k_y + (E_1 A_1/s_1) z_1 \varepsilon_x, \\ M_y &= (D_{22} + E_2 I_2/s_2) k_y + D_{12} k_x + (E_2 A_2/s_2) z_2 \varepsilon_y, \\ M_{xy} &= D_{66} k_{xy}, \end{aligned} \tag{6}$$

where

- A_{ij}, D_{ij} $(i, j = 1, 2$ and $6)$ are extending and bending stiffnesses of the plate without stiffeners,

- E_1, E_2 are the Young modulus of the longitudinal and transversal stiffeners, respectively,

- A_1, A_2 are the section areas of the longitudinal and transversal stiffeners, respectively,

- I_1, I_2 are the inertial moments of cross-section of the longitudinal and transversal stiffeners, respectively,

- s_1, s_2 are the distances between two longitudinal stiffeners and between two transversal stiffeners, respectively,

- z_1, z_2 are the distances from the mid-plane to the centroids of the longitudinal and transversal stiffeners, respectively,

The equilibrium equations of a plate according to [3] are

$$\begin{gathered}
\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \\
\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \\
\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} - q(x, y) = 0.
\end{gathered} \tag{7}$$

Substituting (3) and (6) into (7) after some operations we obtain the equilibrium equations of the multilayered reinforced composite plates

$$\begin{aligned}
&(A_{11} + E_1A_1/s_1)\frac{\partial^2 u}{\partial x^2} + A_{66}\frac{\partial^2 u}{\partial y^2} + (A_{12} + A_{66})\frac{\partial^2 v}{\partial x \partial y} - (E_1A_1/s_1)z_1\frac{\partial^3 w}{\partial x^3} + \\
&\quad + (A_{11} + E_1A_1/s_1)\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + (A_{12} + A_{66})\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x \partial y} + A_{66}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2} = 0, \\
&(A_{22} + E_2A_2/s_2)\frac{\partial^2 v}{\partial y^2} + A_{66}\frac{\partial^2 v}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 u}{\partial x \partial y} - (E_2A_2/s_2)z_2\frac{\partial^3 w}{\partial y^3} + \\
&\quad + (A_{22} + E_2A_2/s_2)\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2} + (A_{12} + A_{66})\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y} + A_{66}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2} = 0, \\
&(D_{11} + E_1I_1/s_1)\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + (D_{22} + E_2I_2/s_2)\frac{\partial^4 w}{\partial y^4} \\
&- (E_1A_1/s_1)z_1\frac{\partial^3 u}{\partial x^3} - (E_2A_2/s_2)z_2\frac{\partial^3 v}{\partial y^3} - (E_1A_1/s_1)z_1\frac{\partial w}{\partial x}\frac{\partial^3 w}{\partial x^3} \\
&- (E_2A_2/s_2)z_2\frac{\partial w}{\partial y}\frac{\partial^3 w}{\partial y^3} - \frac{1}{2}(A_{11} + E_1A_1/s_1)\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}A_{12}\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial x}\right)^2 \\
&- \frac{1}{2}A_{12}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}(A_{22} + E_2A_2/s_2)\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2 - 2A_{66}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x \partial y} \\
&- (A_{11} + E_1A_1/s_1)\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} - 2A_{66}\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - A_{12}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial y^2} - A_{12}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x^2} \\
&- 2A_{66}\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x \partial y} - (A_{22} + E_2A_2/s_2)\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial y^2} - q(x, y) = 0,
\end{aligned} \tag{8}$$

in which $q(x,y)$ is the lateral load, which can be expanded in a double Fourier series

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}. \tag{9}$$

For uniformly distributed load of intensity q_0, the coefficients q_{mn} are given by

$$q_{mn}=\frac{16q_0}{mn\pi^2}\big(-1\big)^{\frac{m+n}{2}},\quad m,n=1,3,5,\dots. \tag{10}$$

If the boundary conditions discussed here can be satified, the displacements are represented by

$$\begin{aligned} u&=U_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,,\\ v&=V_{mn}\sin\frac{n\pi x}{a}\cos\frac{n\pi y}{b}\,,\\ w&=W_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \end{aligned} \tag{11}$$

where

- a, b: edges of plate in x and y axial directions respectively,
- m, n: the numbers of halfwave in the x and y axial directions respectively.

Substituting expressions (11) into the equilibrium equations (8) and applying the Galerkin procedure yield the set of three algebraic equations with respect to the amplitudes U_{mn}, V_{mn}, W_{mn}, where the first two equations of this system are linear algebraic equations for U_{mn}, V_{mn}:

$$\begin{aligned} a_1U_{mn}+a_2V_{mn}&=a_3W_{mn}+a_4W_{mn}^2,\\ a_5U_{mn}+a_6V_{mn}&=a_7W_{mn}+a_8W_{mn}^2. \end{aligned} \tag{12}$$

Getting from (12) expression U_{mn}, V_{mn} with respect to W_{mn} and substituting into the third equation we obtain a non-linear equation with respect to W_{mn}

$$a_9W_{mn}^3+A_{10}W_{mn}^2+a_{11}W_{mn}=q_{mn}, \tag{13}$$

where a_i are coefficients which depend on the material, geometry and the half wave,

$$\begin{aligned} a_1&=(A_{11}+E_1A_1/s_1)\frac{m^2b}{a}+A_{66}\frac{n^2a}{b}\,,\\ a_2&=a_5=(A_{12}+A_{66})mn,\\ a_3&=(E_1A_1/s_1)z_1\frac{m^3\pi b}{a^2}\,,\\ a_4&=-\frac{16}{9}\Big[2(A_{11}+E_1A_1/s_1)\Big(\frac{m}{a}\Big)^2\frac{b}{n\pi}-(A_{12}-A_{66})\frac{n}{b\pi}\Big],\\ a_6&=(A_{22}+E_2A_2/s_2)\frac{n^2a}{b}+A_{66}\frac{m^2b}{a}\,,\\ a_7&=(E_2A_2/s_2)z_2\frac{n^3a\pi}{b^2}\,, \end{aligned}$$

$$
\begin{aligned}
a_8 &= -\frac{16}{9}\Big[2(A_{22}+E_2A_2/s_2)\Big(\frac{n}{b}\Big)^2\frac{a}{m\pi}-(A_{12}-A_{66})\frac{m}{a\pi}\Big],\\
a_9 &= \frac{3}{128}\Big[(A_{11}+E_1A_1/s_1)\frac{m^4b}{a^3}+2\Big(A_{12}+\frac{2}{3}A_{66}\Big)\frac{(mn)^2}{ab}+(A_{22}+E_2A_2/s_2)\frac{n^4a}{b^3}\Big]+\\
&\quad+\frac{H_1(a_6a_4-a_2a_3)+H_2(a_1a_8-a_5a_4)}{a_1a_6-a_2a_5},\\
a_{10} &= \frac{8}{9}\Big[(E_1A_1/s_1)z_1\Big(\frac{m}{a}\Big)^3\frac{b}{n\pi^2}+(E_2A_2/s_2)z_2\Big(\frac{n}{b}\Big)^3\frac{a}{m\pi^2}\Big]\\
&\quad+\frac{H_1(a_3a_6-a_2a_7)+H_2(a_1a_7-a_3a_5)+H_3(a_6a_4-a_2a_8)+H_4(a_1a_8-a_5a_4)}{a_1a_6-a_2a_5},\\
a_{11} &= \frac{1}{4}\Big[(D_{11}+E_1I_1/s_1)\frac{m^4b}{a^3}+2(D_{12}+2D_{66})\frac{(mn)^2}{ab}+(D_{22}+E_2I_2/s_2)\frac{n^4a}{b^3}\Big]\\
&\quad+\frac{H_3(a_3a_6-a_2a_7)+H_4(a_1a_7-a_3a_5)}{a_1a_6-a_2a_5},\\
H_1 &= -\frac{16}{9}\Big[\Big(\frac{m}{a}\Big)^2\frac{b}{n\pi^3}(A_{11}+E_1A_1/s_1)+(A_{12}+2A_{66})\frac{n}{b\pi^3}\Big],\\
H_2 &= -\frac{16}{9}\Big[\Big(\frac{n}{b}\Big)^2\frac{a}{m\pi^3}(A_{22}+E_2A_2/s_2)+(A_{12}+2A_{66})\frac{m}{a\pi^3}\Big],\\
H_3 &= -\frac{1}{4}(E_1A_1/s_1)z_1\frac{m^3b}{a^2\pi},\quad H_4=-\frac{1}{4}(E_2A_2/s_2)z_2\frac{n^3a}{b^2\pi}\cdot
\end{aligned}
$$

4. Finite element method

Based on strain energy principle, the finite element method has built equilibrium equation of the plate [7]

$$\big[K\big]\{q\}=\{F\}. \tag{14}$$

For building equation (14), we need to build matrix $[K]$, which are built from stiffness matrix of element $[K_e]$.

According to [4] for building $[K_e]$, we can see multilayered reinforced composite plates, which are a system of unreinforced plates and beams. From this opinion, the building stiffness matrix $[K_e]$ of reinforced plates is difined

$$[K_e]=[K_e^t]+K_e^d], \tag{15}$$

where: $[K_e^t]$, $[K_e^d]$ are stiffness matrices of the plate and beam elements.

* *Stiffness matrix of the plate elements* $\big[K_e^t\big]$

The relation between deformation and node displacement is of the form

$$\{\varepsilon^t\}=\big[B^t\big]\{q_e\}, \tag{16}$$

where

$$\big[B^t\big]=\big[B_0^t\big]+\big[B_L^t\big]. \tag{17}$$

in which is the same matrix as in linear infinitesimal strain analysis, $\left[B_L^t\right]$ is the large strain matrix depending on $\{q_e\}$.

Thus

$$d\{\varepsilon^t\} = d\big(\left[B^t\right]\{q_e\}\big) = \left[B^t\right]d\{q_e\} + \{q_e\}d\left[B^t\right]. \tag{18}$$

Because $\left[B_L^t\right]$ depends on $\{q_e\}$, $d\left[B^t\right] = d\left[B_L^t\right]$ and $\{q_e\}d\left[B^t\right] = \left[B_L^*\right]d\{q_e\}$, then (18) become

$$d\{\varepsilon\} = \big(\left[B^t\right] + \left[B_L^*\right]\big)d\{q_e\}, \tag{19}$$

where $\left[B_L^*\right]$ has the same form as $d\left[B_L^t\right]$ but instead of dq_i we put q_i

$$d\left[B_L^t\right] = \begin{bmatrix} [0] & d\left[B_{Lu}^t\right] \\ [0] & [0] \end{bmatrix} \qquad \left[B_L^*\right] = \left[B_L\right]. \tag{20}$$

According to [7], the sum of internal and external forces is difined as follows

$$\left[\overline{Q}\right] = \int\limits_S \left[B^t\right]^T \{\overline{\sigma}^t\}dS - \{F\} \tag{21}$$

in which $\{F\}$ - external forces, from (21) we receive

$$d\{\overline{Q}\} = \int\limits_S d\left[B^t\right]^T\{\overline{\sigma}^t\}dS + \int\limits_S \left[B^t\right]^T d\{\overline{\sigma}^t\}dS. \tag{22}$$

Otherwise, from (1) we obtain

$$d\{\overline{\sigma}^t\} = \left[\overline{D}^t\right]d\{\varepsilon_t\} - \left[\overline{D}^t\right]\Big(\left[B^t\right] + \left[B_L^*\right]\Big)f\{q_e\}. \tag{23}$$

Substituting (23) into (22) yields

$$d\{\overline{Q}\} = \int\limits_S Dd\left[B^t\right]^T\{\overline{\sigma}^t\}dS + \int\limits_S \left[B^t\right]\left[\overline{D}^t\right]\Big(\left[B^t\right] + \left[B_L^*\right]\Big)dSd\{q_e\}. \tag{24}$$

Because $d\left[B^t\right]^T = d\left[B_{Lu}^t\right]^T$ and $\left[B_L^*\right] = \left[B_L^t\right]$, one can get

$$d\{\overline{Q}\} = \int\limits_S d\left[B_{Lu}^t\right]^T\{\overline{\sigma}^t\}dS + \left[\overline{K}\right], \tag{25}$$

in which

$$\left[\overline{K}\right] = \int\limits_S \Big(\left[B^t\right]^T\left[\overline{D}^t\right]\left[B^t\right] + \left[B^t\right]^T\left[\overline{D}^t\right]\left[B_L^T\right]\Big)dS. \tag{26}$$

Substituting $\left[B_t\right]$ from (17) into (26) and after some operations we obtain

$$\left[\overline{K}\right] = \left[K_{0e}^t\right] + \left[K_{Le}^t\right], \tag{27}$$

where $\left[K^t_{0e}\right]$ is the same stiffness matrix as in linear infinitesimal strain analysis. For elements of the plate

$$\left[K^t_{0e}\right] = \int_S \left[B^t_0\right]^T \left[\overline{D}^t\right] \left[B^t_0\right] dS. \tag{28}$$

Matrix $\left[K^t_{Le}\right]$ is the large displacement matrix, which can be defined as follows

$$\left[K^t_{Le}\right] = \int_S \Big(2\left[B^t_0\right]^T \left[\overline{D}F^t\right] \left[B^t_L\right] + \left[B^t_L\right]^T \left[\overline{D}^t\right] \left[B^t_0\right] + 2\left[B^t_L\right]^T \left[\overline{D}^t\right] \left[B^t_L\right]\Big) dS. \tag{29}$$

The first term of equation (25) can generally be written as:

$$\int_S d\left[B^t\right]\left\{\overline{\sigma}^t\right\} dS = \left[K^t_{\sigma e}\right] d\{q_e\} \tag{30}$$

where $\left[K^t_{\sigma e}\right]$ is a symmetric matrix which dependens on the stress level. This matrix is known as initial stress matrix or geometric matrix.

According to [7] we have

$$\left[K^t_{\sigma e}\right] = \begin{bmatrix} [0] & [0] \\ [0] & \left[K^u_{\sigma e}\right] \end{bmatrix},$$

with

$$\left[K^u_{\sigma e}\right] = \int_S \left[G^t\right]^T \begin{bmatrix} N^t_x & N^t_{xy} \\ N^t_{xy} & N^t_y \end{bmatrix} \left[G^t\right] dS, \tag{31}$$

in which

$$\left[G^t\right] = \begin{bmatrix} \dfrac{\partial N^t_{u_1}}{\partial x} & \dfrac{\partial N^t_{u_2}}{\partial x} & \dots & \dots & \dfrac{\partial N^t_{u_{11}}}{\partial x} & \dfrac{\partial N^t_{u_{12}}}{\partial x} \\ \dfrac{\partial N^t_{u_1}}{\partial y} & \dfrac{\partial N^t_{u_2}}{\partial y} & \dots & \dots & \dfrac{\partial N^t_{u_{11}}}{\partial y} & \dfrac{\partial N^t_{u_{12}}}{\partial y} \end{bmatrix} \tag{32}$$

Thus, for element of the plate we obtain

$$d\left[\overline{Q}\right] = \Big(\left[K^t_{0e}\right] + \left[K^t_{Le}\right] + \left[K^t_{\sigma e}\right]\Big) d\{q_e\} = \left[K^t_e\right] d\{q_e\}$$

and stiffness matrix of the element of the plate

$$\left[K^t_e\right] = \left[K^t_{0e}\right] + \left[K^t_{Le}\right] + \left[K^t_{\sigma e}\right]. \tag{33}$$

** Stiffness matrix of element of the beam* $\left[K^t_{\sigma e}\right]$

Using two-noded element of the beam with three degree of freedom at each node

$$\left\{u^d_1, w^d_1, \varphi^d_1, u^d_2, w^d_2, \varphi^d_2\right\}^T$$

Acording to [8] for a element of multilayered composite beam, which works in the elastic state the relation between internal force and deformation are of the form

$$\{\sigma^d\} = \left[\overline{D}^d\right]\{\varepsilon^d\}, \tag{34}$$

in which

$$\{\sigma^d\} = \{N_x^d \quad M_y^d\}^T; \quad \left[\overline{D}^d\right] = \begin{bmatrix} [A^d] & [B^d] \\ [B^d] & [D^d] \end{bmatrix} \tag{35}$$

The matrices $[A^d]$, $[B^d]$, $[D^d]$ are defined in [8], $\{\varepsilon^d\}$ the deformation of point of the middle surface

$$\{\varepsilon^d\} = \left\{\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 \quad -\frac{d^2w}{dx^2}\right\}^T \tag{36}$$

or we can be rewritten in the form

$$\{\varepsilon^d\} = \left\{\begin{matrix} \{\varepsilon_{0m}^d\} \\ \{\varepsilon_{0u}^d\} \end{matrix}\right\} + \left\{\begin{matrix} \{\varepsilon_L^d\} \\ \{0\} \end{matrix}\right\} \tag{37}$$

Expressing the defomation with noded diplacement as follows

$$\{\varepsilon^d\} = [B^d]\{q_e^d\}, \tag{38}$$

where

$$[B^d] = [B_0^d] + [B_L^d]. \tag{39}$$

Similar to the multilayered composite plate, we obtain stiffness matrix of element of beam as follow

$$[K_e^d] = [K_{0e}^d] + [K_{Le}^d] + [K_{\sigma e}^d], \tag{40}$$

where

$$\begin{aligned} [K_{0e}^d] &= \int_L [B_0^d]^T \left[\overline{D}_d\right] [B_0^d] dx \\ [K_{Le}^d] &= \int_L \Big(2[B_0^d]^T \left[\overline{D}_d\right] [B_L^d] + [B_L^d] \left[\overline{D}_d\right] [B_0^d] + 2[B_L^d] \left[\overline{D}_d\right] [B_L^d]\Big) dx \\ [K_{\sigma e}^d] &= \int_L [G^d]^T [N^d] [G^d] dx. \end{aligned} \tag{41}$$

5. Numerical examples

Let's consider a simply supported stiffened rectangular symmetrical composite plate: $a = 0.8\,\mathrm{m}$; $b = 0.5\,\mathrm{m}$. The materials of the plate are composed by Thornel 300 graphite fibers and Narmco 5205 Thermosetting Epoxy resin [5], the properties of these materials are: $E_1 = 127.4\,\mathrm{GPa}$; $E_2 = 13\,\mathrm{GPa}$; $G_{12} = 6.4\,\mathrm{GPa}$; $\nu_{12} = 0.38$; The plate has six layers: $[45/-45/90/90/-45/45]$; thickness of each layer: $t = 0.5\,\mathrm{mm}$; The laminate plate is reinforced by longitudinal and transversal stiffeners, which were made of CPS material, the stiffeners have the same sizes, as follows: $b_g \times h_g = 4\,mm \times 6\,mm$; Spacing of longitudinal and transverse stiffeners is: $s_1 = s_2 = 0.1\,\mathrm{m}$.

The results according to two methods are presented on the Figs 2, 3, 4.

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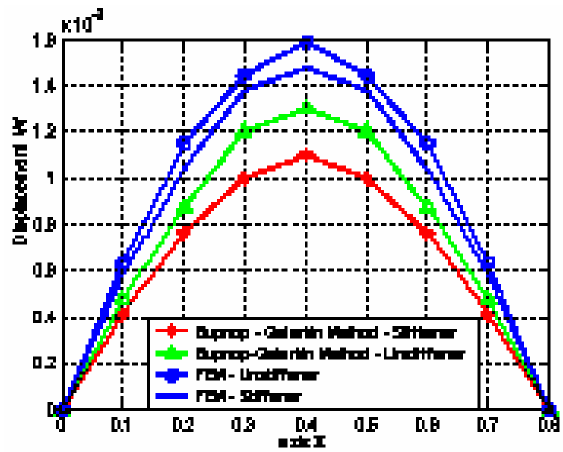

Fig. 2. Displacement of cut trace, going over the center of plate and paralled with x axis

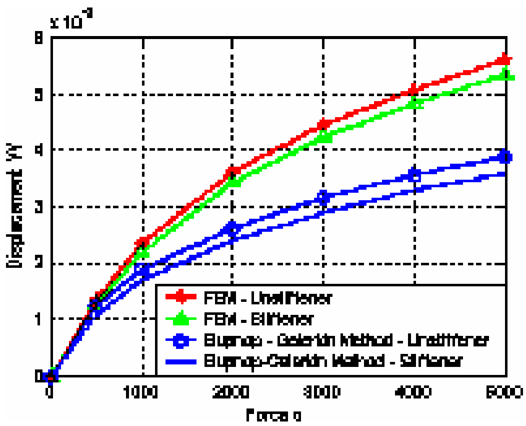

Fig. 3, Relation between displacement and external force

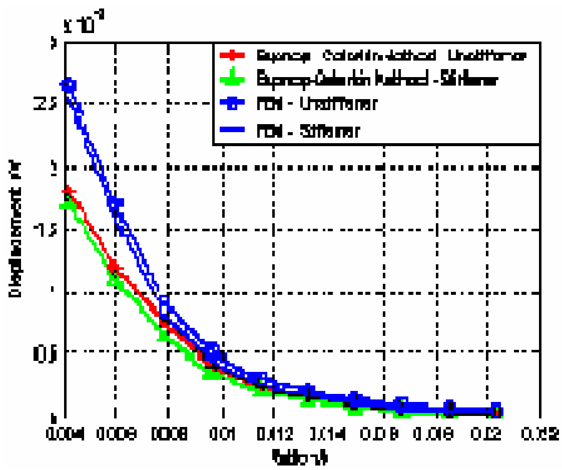

Fig. 4. Effecty of thickness of the plate

Conclusions

The results by the Bupnov - Galerkin method agree qualitatively with those by the Finite element method, but the results by the Bupnov - Galerkin method are smaller than that by the FEM. This difference can be reduced, if we take more number of terms in the double Fourier series of the displacements.

Displacement of the non-linear analysis of multilayered reinforced CPS plates are smaller than that of multilayered unreinforced CPS plates. This means, the hardness of multilayered reinforced CPS plates is bigger than that of multilayered unreinforced CPS plates.

Displacement and stress of the linear analysis of multilayered CPS plate are directly proportional to external force, but displacement and stress of the non-linear analysis of multilayered CPS plate aren't direct by proportional to external force. If external forces are small, displacement in non-linear problem is approximately equal with linear displacement. When external force increases, the difference between linear and non-linear analysis also gets increased. This means non-linear analysis is exacter than linear analysis.

If the thickness of the plate is increased, the difference between reinforced and unreinforced plate also gets reduced, so the stiffener takes effect for thin plates.

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References

1. Tran Ich Thinh, *Composite materials - mechanics and calculation of structures.* Ed. Education, (1994) (in Vietnamese).
2. X. P. Timosenko., J. M. Gere, *Theory of elastic stability,* Science and Technical Publisher, (1976) (in Vietnamese).
3. M. W. Hyer, *Stress analysis of fiber reinforced composite materials.* Mc. Graw. Hill. International Editions, (1998).
4. M. Kolli, K. Chandrashekhara, Nonlinear static and dynamic analysis of stiffened laminated plates, *Int. J. Non-linear Mechanics,* Vol.32, **No1**(1997) pp. 89-101.
5. Victor Birman, Theory and comparision of the effect of composite and shape memory alloy stiffeners stability of composite shells and plates. *Int. J. Mech. Sci.,* Vol.39, **No10**, pp.1139-1149.
7. Dao Huy Bich, Non-linear analysis of laminated plates, *Vietnam Journal of Mechanics, NCNST of Vietnam,* Vol. 24, **No 4**(2002) pp. 197-208.
7. O. C. Zienkiewic, *The finite element method in engineering sciences,* Maidenhead, M. C. Graw Hill Publishing Co. Ltd. 1975.
8. Hoang Xuan Luong, Pham Tien Dat, Mechanics of composite materials, *Military Technical Academy,* Hanoi 2003.