

An approach for obtaining approximate formulas for the Rayleigh wave velocity

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Abstract: In this paper, we introduce an approach for finding analytical approximate formulas for the Rayleigh wave velocity for isotropic elastic solids and anisotropic elastic media as well. The approach is based on the least-square principle. To demonstrate its application, we applied it in order to obtain an explanation for Bergmann's approximation, the earliest known approximation of the Rayleigh wave velocity for isotropic elastic solids, and used it to establish a new approximation. By employing this approach, the best approximate polynomials of the second order of the cubic power and the quartic power in the interval $[0, 1]$ were found. By using the best approximate polynomial of the second order of the cubic power, we derived an approximate formula for the Rayleigh wave speed in isotropic elastic solids which is slightly better than the one given recently by Rahman and Michelitsch by employing Lanczos's approximation. Also by using this second order polynomial, analytical approximate expressions for orthotropic, incompressible and compressible elastic solids were found. For incompressible case, it is shown that the approximation is comparable with Rahman and Michelitsch's approximation, while for the compressible case, it is shown that our approximate formulas are more accurate than Mozhaev's ones. Remarkably, by using the best approximate polynomials of the second order of the cubic power and the quartic power in the interval $[0, 1]$, we derived an approximate formula of the Rayleigh wave velocity in incompressible monoclinic materials, where the explicit exact formulas of the Rayleigh wave velocity so far are not available. ?? 2007 Elsevier B.V. All rights reserved.

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