INFLUENCE OF RELATIVE DENSITY ON EFFECTIVE STRESS CHANGE OF SATURATED GRANULAR MATERIALS UNDER MULTI-DIRECTIONAL CYCLIC SIMPLE SHEAR

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ABSTRACT: This paper presents the results of cyclic simple shear test under constant the volume condition in order to investigate the effect of density on effective stress change of saturated granular materials. Several series of multidirectional cyclic simple shear test were performed on Toyoura sand and the granulated blast furnace slag (GBFS) with different relative densities under the phase difference $\theta = 90^{\circ}$ (gyratory cyclic shearing) with various cyclic shear strain amplitudes. From the test results, it was found that the effective stress change of granular materials was influenced dominantly by cyclic shear strain amplitude. It was also clarified that density has a significant effect on the effective stress change of granular materials. Finally, a simple method is presented for estimating the effective stress change of saturated granular materials with different relative densities. The validity of this method is confirmed by comparing with test results under multi-directional cyclic simple shear conditions.

INTRODUCTION

In saturated granular materials, as cyclic loading occurs, the pore water pressure increases and results in a reduction of effective stress. The term of liquefaction has been defined as the state at which the pore water pressure increases until it (or nearly) equals the total vertical stress. The reduction of effective stress allows the soil to deform (Lee and Seed 1967; Seed 1979).

Because the shear strength of granular materials is primarily due to frictional forces, it is almost proportional to the packing density condition of particles (Youd 1977). Relative density has been recognized as a dominant factor influencing the cyclic strength of saturated granular soils. Tatsuoka et al. (1986) performed undrained cyclic triaxial tests on Toyoura sand and found that the cyclic undrained strength tends to increase proportionally with the relative density; however, for the relative density higher than 70%, this cyclic strength becomes increase sharply. By using simple shear test, Vaid and Sivathayalan (1996) showed the relationship between the cyclic resistance and relative density on Fraser Delta sand. It is found that the cyclic resistance increases with increasing relative density at all levels of confining stress; however, the cyclic resistance of loose sands is independent of the confining stress level. The same trend is observed in the variation of cyclic resistance with relative density and confining stress in cyclic triaxial tests (Vaid and Thomas 1995). Further, Sivathayalan and Vaid (2004) showed that relative density plays an important role on the post-liquefaction strength of sand; denser sands reach higher strength with small strain development. It is

also recognized that the strength develops at a faster rate in triaxial compression test than in simple shear or triaxial extension test.

Mostly, however, the previous investigations have only considered in the case of uni-directional cyclic shear conditions. In fact, the shear strain during earthquakes shows multi-directional paths as shown in Fig. 1. Matsuda et al. (2004) performed multi-directional cyclic shear tests on saturated granular materials under constant volume condition, and found that effective stress reduction in the case of multi-directional shear is larger than that of uni-directional shear. However, the influence of density on effective stress reduction during multi-directional cyclic shear is still not clarified. Therefore, the purpose of this study is to investigate the influence of relative density on effective stress change of saturated granular materials under multi-directional cyclic simple shear conditions.

In this study, several series of multi-directional cyclic simple shear test under constant volume condition were performed on Toyoura sand and the granulated blast furnace slag (GBFS) with different relative densities. The cyclic loading tests were conducted under the phase difference $\theta = 90^{\circ}$ with various cyclic shear strain amplitudes. The cyclic strain model proposed by Fukutake and Matsuoka (1989) was adopted in order to clarify the pattern of effective stress change of saturated granular materials. Then, a simple method is presented for estimating the effective stress change of saturated granular materials with different relative densities. The validity of this method is finally confirmed by comparing with the test results.



Figure 1 The orbit of shear strain in the ground measured at Hyogo-ken Nanbu Earthquake, January 17, 1995 (Matsuda et al., 2004)



Figure 2 The multi-directional cyclic simple shear test apparatus

MULTI-DIRECTIONAL CYCLIC SIMPLE SHEAR TEST UNDER CONSTANT VOLUME CONDITION

Basic Properties of Apparatus

The outline of the multi-directional cyclic simple shear test apparatus is shown in Fig. 2. Having the Kjellman type shear box, this apparatus can give any types of cyclic displacement to the bottom of specimen from two perpendicular directions. For each test, a predetermined phase difference θ for waves of X and Y directions can be applied. When the phase difference θ is 0°, it is known as uni-directional cyclic shear condition, whereas in a case of multi-directional cyclic shear test, the phase difference θ is 90° (or $\pi/2$ in *radian*). For each test, a predetermined vertical stress can be applied to the specimen by the electric-controlled aero-servo system. As a strain-controlled test, the shear strain amplitude γ can be obtained as a ratio between the horizontal displacement δ and the initial height of the specimen.

Figure 3(a) shows the shear wave form during unidirectional cyclic test, where the shear strain is applied only in one direction (in this case: X direction). Figure 3(b) shows the shear wave for multi-directional shear, in which shear strain is applied to both X and Y directions.

Samples and Methods

One of the test materials used was Toyoura sand (specific gravity $G_{\rm s} = 2.637$, maximum void ratio $e_{\rm max} = 0.991$, minimum void ratio $e_{\rm min} = 0.630$). The other test materials were the granulated blast furnace slag ($G_{\rm s} = 2.643$, $e_{\rm max} = 1.510$, $e_{\rm min} = 1.033$). The gradation curves of these materials are shown in Fig. 4. It is observed that the granulated blast furnace slag (hereinafter referred to as GBFS) has almost the similar unit weight of soil grains, but



Figure 3 Cyclic shear waves



Figure 4 The gradation curves of samples

coarser than that of Toyoura sand. Recently in Japan, GBFS is considered to be one of promising materials in geotechnical engineering for its particular properties: light weight, high shear strength and high permeability. Because of these properties, the report for using GBFS as a useful material has already been published (JGS 2010).

In the shear box, the specimen was covered with the rubber membrane, which is placed in contact with the inner side of 10 acrylic rings which are stacked. Each acrylic ring has 2.0 mm in thickness. By this arrangement, the specimen is 75 mm in diameter and 20 mm in height, and the specimen is prevented from lateral swelling but permitted the shear deformation. Throughout the cyclic shearing, the vertical displacement was restricted to keep the volume of specimen constant.

The specimen was prepared as follows: saturated granular materials were poured into the shear box with predetermined relative density Dr, and then the specimens were consolidated for 15 minutes under the vertical stress σ_v ' = 49 kPa. In this study, the specimens having relative densities Dr = 50%, 70% and 90% were prepared in order to investigate the effect of density on effective stress change of granular materials under multi-directional cyclic shear condition ($\theta = 90^\circ$). The wave form of applied shear strain is sinusoidal with period T = 2.0 s, and the shear strain amplitude was set as a range from $\gamma = 0.1\%$ to 1.0% and the number of cycles *n* was set in a range from n = 1 to 150.

Cyclic shear strain parameters

Fukutake and Matsuoka (1989) proposed the model that can be simulate the strain paths of granular materials under drained multi-directional cyclic shear conditions. In this model, the shear strain path on the horizontal plane can be represented as the resultant shear strain Γ , which shows the radial distance from the origin, and the cumulative shear strain G^* , which shows the length along the shear strain path. In this study, the benefits of these parameters were adopted to investigate the effective stress change of granular materials under undrained cyclic shear conditions. The resultant shear strain Γ and the cumulative shear strain G^* are expressed as follows:

$$G^* = \Sigma \Delta G^* = \Sigma \sqrt{\Delta \gamma_X^2 + \Delta \gamma_Y^2}$$
(1)

$$\Gamma = \sqrt{\Delta \gamma_X^2 + \Delta \gamma_Y^2} \tag{2}$$

Where, $\Delta \gamma_X$ and $\Delta \gamma_Y$ represent the shear strain increments at *X* direction and *Y* direction on the horizontal plane, respectively.

TEST RESULTS AND DISCUSSIONS

Effective Stress Change of Saturated Granular Materials Induced by Multi-Directional Shearing

It has been shown (Matsuda and Nagira 2000; Tamada et al., 2008) that the relationship between effective stress reduction and shear strain amplitude subjected to unidirectional cyclic shear under undrained condition on saturated clays can be formulated by a hyperbola. Derived from their result, the relationship between the effective stress reduction ratio $|\Delta \sigma'_v / \sigma'_{vo}|$ and the cumulative shear strain G^* on saturated granular materials subjected to multidirectional shear can be expressed as follows:

$$\left|\frac{\Delta\sigma_{\nu}^{'}}{\sigma_{\nu0}^{'}}\right| = \frac{G^{*}}{\alpha + \beta \bullet G^{*}}$$
(3)

Where α and β can be defined as:

$$\alpha = A \bullet \gamma^m \tag{4}$$

$$\beta = \frac{\gamma}{B + C \cdot \gamma} \tag{5}$$

The parameters of A, B, C and m in Eqs. (4) and (5) can be obtained empirically by the curve-fitting method.

Figure 5 shows the relationships between cumulative shear strain G^* and the effective stress reduction ratio $|\Delta \sigma'_v / \sigma'_{vo}|$ on Toyoura sand with different relative densities, in which $\Delta \sigma'_{\rm v}$ is defined as the decrease of effective vertical stress and σ'_{vo} denotes the initial effective vertical stress. These results were obtained under $\theta = 90^{\circ}$ shearing with different shear strain amplitudes. Also plotted in Fig. 5, the solid curves showing the data that calculated by Eqs. (3) to (5). The parameters A, B, C and m used for Toyoura sand and GBFS were taken from Tables 1 and 2, respectively. From the test results, it is clarified that effective stress change of granular materials was influenced dominantly by cyclic shear strain amplitude; larger shear strain amplitude implicates the sudden decrease of effective vertical stress. However, it is found that at loose density specimen (Dr =50%), the effect of shear strain amplitude is small. It is also noted that at loose and medium density specimens, at higher cumulative shear strain ($G^* \ge 20\%$), the effect of shear strain amplitude on the relationship between $|\varDelta\sigma'_v/\sigma'_{vo}|$ and G^* can be negligible. In addition, density also has a significant effect on the rate of effective stress change of granular materials. At the dense specimens, effective stress decreases slower than that of loose specimens. Similar tendency has been reported from many



Figure 5 Relationship between the cumulative shear strain and effective stress reduction ratio on Toyoura sand with different relative densities

investigations on the cyclic strength of saturated granular materials (e.g., Tatsuoka et al., 1986; Sivathayalan and Vaid, 2004). According to Fig. 5, the curve developed from



GBFS

θ = 90°

15

20

GBFS

 $D_{\rm r} = 70\%$

 $\theta = 90^{\circ}$

15

GBFS

 $D_{\rm r} = 90\%$

20

 $\theta = 90^{\circ}$

20

 $D_{\rm r} = 50\%$

calculated data agrees with the experimental result, thus, the validity of these proposed equations can be confirmed



(a) Relationship between A and Dr



(b) Relationship between B and Dr



(c) Relationship between C and Dr



(d) Relationship between *m* and *D*r

Figure 7 Parameters for Toyoura sand

Table 1 Parameters A, B, C, m for Toyoura sand

Dr (%)	Α	В	С	m
50	1.0	0.03	0.9	-0.5
70	1.2	0.02	1.05	-0.55
90	1.8	0.001	1.03	-0.7

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(a) Relationship between A and Dr



(b) Relationship between B and Dr



(c) Relationship between C and Dr



(d) Relationship between m and Dr

Figure 8 Parameters for GBFS

Table 2 Parameters A, B, C, m for GBFS

Dr (%)	А	В	С	m
50	2.4	-0.03	1.0	-0.5
70	2.5	-0.044	1.1	-0.56
90	3.0	-0.05	1.15	-0.6

Figure 6 shows the relationships between G^* and $|\Delta \sigma' \sqrt{\sigma'_{vo}}|$ on GBFS with different relative densities. It can be clarified that the rate of effective stress reduction on GBFS samples are lower than that of Toyoura sand. The similar tendency of this behavior has been reported by Matsuda et al. (2004). From this figure, it is also observed that the experimental results nearly correspond to the calculated values. However, the discrepancies between the experimental and calculated values were observed on medium (Dr = 70%) and dense specimens (Dr = 90%), when subjected to larger shear strain amplitude. This result was due possibly the effects of crushable properties of GBFS particles.

The relationships between relative densities and the parameters of *A*, *B*, *C* and *m* for Toyoura sand and GBFS are shown in Figs. 7 and 8, respectively. It is seen from these figures that those parameters can be evaluated as a linear function of relative density. This means that the effective stress change of saturated granular materials can be estimated from this proposed model, at least, for saturated granular materials under multi-directional cyclic shear having $\theta = 90^{\circ}$.

CONCLUSIONS

In this study, several series of multi-directional cyclic simple shear test under constant volume condition were performed on Toyoura sand and GBFS. The main conclusions are summarized as follows.

- (1) Shear strain amplitude has a significant effect on effective stress change of granular materials; however, at specimens of loose density, the effect of shear strain amplitude is small.
- (2) For loose and medium density specimens, at higher cumulative shear strain (G* ≥ 20%), the effect of shear strain amplitude on effective stress change of granular materials becomes negligible.
- (3) At the dense specimens, effective stress decreases with a slower rate than that of specimens having lower density. The rates of effective stress reduction on GBFS samples are lower than that of Toyoura sand.
- (4) Effective stress change of saturated granular materials can be estimated from the proposed model. Furthermore, the parameter that used in this model is a function of relative density of saturated granular materials.

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