LANDSLIDE HAZARD: A COUPLED HYDRO-MECHANICAL ANALYSIS WITH VARIATION OF SUCTION AND SOIL STIFFNESS DURING WETTING PROCESS - MA RIVER CASE STUDY

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ABSTRACT: In this study a coupled hydro-mechanical analysis using FEM method was applied to simulate numerically the landslide process of the Ma river bank. The elasto-viscoplastic model was employed to investigate the unsaturated soil behaviour based on the Barcelona Basic Model (BBM-VP). The model allows taking into account the hardening and softening due to mechanical and hydraulic loading. For hydraulic behaviour, generalized Darcy's law is used with porosity dependant intrinsic permeability and employing a van Genuchten type relationship between suction and degree of saturation. For evaluation of the results obtained via the BBM-VP constitutive law, the landslide process was simulated employing two different material laws, and namely, the linear elastic viscoplastic Drücker-Prager type model accounting for suction and the linear elastic-perfect plastic Mohr-Coulomb model without considering the effect of soil suction. The evolution of displacements and the inelastic deformation during infiltration process are discussed based on the simulation of dry to rainy season water level change. We tried to assess the potential of a coupled hydro-mechanical model accounting for time dependent effects for quantifying stresses and deformations necessary to trigger slope failure.

INTRODUCTION

Landslides are well recognized to cause significant damage to the natural and built environment. With the effects of global warming, the increase of sea level, and the change of hydrography, there is high risk in triggering dike and riverbank instability processes.

Within three years, there have been many landslides along the bank of the Ma River, Vietnam (

Figure 1). For that, several reasons have been pointed out: featured geology profiles, the properties of the soil layers, the specific topography-geomorphology, the particularity of the hydrography profile, the characteristics of the flow in seasons, and human activities influencing on the geo-environment (Trần et al. 2007). The causes of landslide or earth topples in Ma River valley need to be analyzed for each particular case. In this study, the phenomenon is analyzed within the framework of soil mechanics for partially saturated soil.

The most used uncoupled method to treat slope stability problems is the limit equilibrium approach. The stresses and strains are related by a material law and a fixed pore-pressure field is used in the uncoupled slope stability analysis. The factors of safety can be afterwards calculated via e.g. the methods of slices (Bishop's simplified method, Ordinary method of slices) or the mass methods (Culmann's method; Fellenius−Taylor method). Commonly used material models presently are Mohr-Coulomb model, Hardening Soil model, Soft Soil model, Cam Clay model etc. However, such uncoupled models do not allow simulating reduction of shear strength and the hardening or softening behaviour due to wetting and drying loading cycles that may be of paramount importance for the slope stability.

Regarding the Ma riverbank, most of it is composed of clayey soil (Phạm et al. 2008). Several concrete walls were built after the river flowed through the cities. Water level is low in dry season, therefore the soil composing the levee's body starts drying and its pore system becomes filled with water and air. Water level increases in the rainy season and this may be a possible trigger of slope instability. An increase of water level to the levee base causes softening decreasing breaking the bonds created by surface tension between the soil particles. The water is sucked up from the levee due to the suction, thus producing higher water content or higher bulk density of the soil and therefore the slope becomes imposed to greater body force. This phenomenon induces an increase of settlement at the levee footing and it could be a reason for instability of the levee body. In order to account for these facts the behaviour of the soil must be simulated by means of unsaturated soil mechanics. In this paper, we quantify the levee deformation induced by the water level increase and address the failure potential of the riverbank using finite element modelling that couples solid deformation with suction in an unsaturated soil. For comparison, the process of slope deformation due to footing and seasonal water level change is analysed via three different models accounting for soil suction to different extend or ignoring it.

For the numerical simulation of the season induced hydraulic loading and landslide process, we use the coupled hydro-mechanical finite element program CODE_BRIGHT for unsaturated soil (Olivella et al. 1996, 2000). For the mechanical behaviour, elasto-viscoplastic model based on Barcelona Basic Model (BBM-VP) is utilized (Alonso et al., 1990, 2005; Collin et al. 2002, 2008 and Datcheva et al., 2005). The modified Darcy's law taking into account the relationship between suction and degree of saturation is used for modelling the hydraulic behaviour. The BBM allows simulating the hardening and softening behaviour due to loading und unloading process or wetting and drying process as well as shear strength by applying critical state line. The capillary pressure in unsaturated soil is also taken into account by describing through the van Genuchten model (van Genuchten, 1980) the relationship between suction and degree of saturation as well as the conductivity in the Darcy's law as a function of the degree of saturation. In sum, the coupled hydro-mechanical model allows to simulate numerically the behaviour of the soil involved in riverbank problems. The calculations in this paper are performed using two finite element programs, namely CODE_BRIGHT (DIT-UPC, 2009) and Plaxis (Brinkgreve et al., 2008).

The reliability of the numerical model needs to be assessed. Therefore in addition to the BBM-VP model, two other models are used, namely a Perzyna type linear elastic viscoplastic model based on the Drücker-Prager failure criterion (DP-VP) and a linear elastic - perfect plastic model (MC) based on the Mohr-Coulomb failure criterion. In the simulation with DP-VP model, the hydraulic equations are also solved but suction does not influence the parameters involved in the mechanical model. The simulation with MC model is performed using different finite element program than CODE_BRIGHT, viz, Plaxis (Brinkgreve et al., 2008). In this later case, only the saturated flow is considered, i.e. the hydraulic properties are not dependent on suction.

Figure 1 Ma River bank landslide examples

FORMULATION OF THE CONSTITUTIVE MODELS

Mechanical constitutive model

The models considered here are elasto-viscoplastic models for unsaturated soil based on the Perzyna viscoplasticity, Perzyna (1966). The formulation for the constitutive modelling is based on the use of pair stress state variables, namely net stress and suction. Other alternative approach exists, e.g. defining effective stress and introducing suction dependent constitutive functions, Borja&White, 2010. Suction *(s)* is the difference between gas pressure P_g and liquid pressure P_l. Suction is zero when the soil pore system is filled with only one fluid. Therefore, the stress variables are suction and net stress for unsaturated condition and effective stress in saturated state. The definition of the stress variables in both saturated and unsaturated state is given via Eq.(1):

$$
\begin{aligned}
s &= \max\left[\left(P_g - P_l\right); 0\right] \\
\boldsymbol{\sigma}' &= \boldsymbol{\sigma}^{total} - \max(P_g; P_l)\mathbf{I} \\
p' &= p^{total} - \max(P_g; P_l)
\end{aligned}
\tag{1}
$$

where $\boldsymbol{\sigma}^{total}$ is the total stress, $\mathbf{I}$ is identity matrix, p^{total} is the total mean stress, $\boldsymbol{\sigma}'$ and p' are correspondingly the net stress and the mean net stress in unsaturated case or the effective stress and the effective mean stress in saturated case.

The two viscoplastic models considered here are implemented in the FE program CODE_BRIGHT (DIT-UPC, 2009). The first one is a linear elastic viscoplastic model for unsaturated soil based on the BBM (Alonso et al., 2005). The second model is a linear elastic viscoplastic model based on the Drücker-Prager failure criterion. Following the Perzyna visco-plastic concept, the total strain rate is assumed to be a sum of the elastic ($\dot{\varepsilon}^e$) and viscoplastic ($\dot{\varepsilon}^{vp}$) strain rates:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{vp} \tag{2}$$

The elastic part is related to the net stress $\dot{\sigma}'$ through the generalized Hooke's law:

$$\dot{\sigma}' = \boldsymbol{C}^e \dot{\varepsilon}^e \tag{3}$$

where $\boldsymbol{C}^e$ is the elastic stiffness tensor. As proposed in Perzyna (1966), the visco-plastic strain rate is defined as:

$$\dot{\varepsilon}^{vp} = \Gamma \langle \Phi(F) \rangle \frac{\partial G}{\partial \sigma'}$$

$$\Phi(F) = \begin{cases} (F/F_0)^N, & if \quad F > 0 \\ 0, & if \quad F \le 0 \end{cases} \tag{4}$$

where Γ is the viscosity parameter, $\Phi(F)$ is a flow function, F is the yield function, F_o is a normalizing constant in the same units as F. Finally, G is the viscoplastic potential.

a) BBM-VP model

In BBM-VP model F and G in Eq. (4) are given according to Eqs (5) and (6) (Alonso et al., 2005):

$$F(q,p,s) = a\frac{1}{3}q^2 - M^2\gamma\left(p' + p_s\right)\left(p_o - p'\right) \tag{5}$$

where M is the slope of the critical state line and may depend on suction, p_s is the tensile stress limit that follows a linear relationship with suction Eq. (9), p_o is the pre-consolidation pressure depending on suction according to Eq. (7), q is the deviatoric stress, γ and a are model parameters.

The viscoplastic potential in this case reads:

$$G(q,p,s) = a\frac{1}{3}q^2 - \alpha M^2\gamma\left(p' + p_s\right)\left(p_o - p'\right) \tag{6}$$

where α is the non-associativity parameter.

According to Alonso (1990) the pre-consolidation pressure depends on suction in the following way:

$$p_o = p^c \left(\frac{p_o^*}{p^c}\right)^{\frac{\lambda(0)-\kappa}{\lambda(s)-\kappa}} \tag{7}$$

where p^c is a reference pressure, p_o^* is the pre-consolidation pressure for a saturated state, κ and $\lambda(0)$ are model parameters The stiffness parameter for changes in the net mean stress at a given suction (s) is defined by:

$$\lambda(s) = \lambda(0)\left[\left(1-r\right)\exp\left(-\beta s\right) + r\right] \tag{8}$$

where r is a parameter defining the soil stiffness when suction reaches infinity, β is a parameter controlling the rate of increase of soil stiffness with suction.

The tensile strength p_s depends on suction via:

$$p_s\left(s\right) = k\, s \tag{9}$$

where k is a parameter that takes into account the increase of tensile strength due to suction.

M determines the slope of the critical state line and it depends on suction according to (Alonso et al., 2005):

$$M\left(s\right) = M_{dry} - \left(M_{dry} - M_{sat}\right)\left(\frac{M_{sat}}{M_{dry}}\right)^s; \quad \left(M_{sat} < M_{dry}\right) \tag{10}$$

where M_{dry} and M_{sat} are critical state line slopes at dry state and saturated state correspondingly.

b) DP-VP model

In DP-VP model, F and G in Eq. (4) are given by Eq. (11):

$$G = F = q - Mp' - c'\beta_c \tag{11}$$

In this case M and β_c are calculated by Eq. (12) to provide the best fit to the Mohr-Coulomb hexagon and φ' and c' are the effective angle of friction and the cohesion defining the Mohr-Coulomb failure envelope at saturated condition.

$$M = \frac{6\sin\varphi'}{3-\sin\varphi'} \qquad \beta_c = \frac{6\cos\varphi'}{3-\sin\varphi'} \tag{12}$$

c) MC model in Plaxis

According to Plaxis manual, the Mohr-Coulomb yield surface consists of six yield functions when formulated in terms of principle stresses in three dimensions:

$$F_1 = \pm\frac{1}{2}(\sigma'_2-\sigma'_3)-\frac{1}{2}(\sigma'_2+\sigma'_3)\sin\varphi' - c'\cos\varphi'$$
$$F_2 = \pm\frac{1}{2}(\sigma'_3-\sigma'_1)-\frac{1}{2}(\sigma'_3+\sigma'_1)\sin\varphi' - c'\cos\varphi' \quad (13)$$
$$F_3 = \pm\frac{1}{2}(\sigma'_1-\sigma'_2)-\frac{1}{2}(\sigma'_1+\sigma'_2)\sin\varphi' - c'\cos\varphi'$$

where σ'_1, σ'_2, σ'_3 are the principle effective stresses. For more explanation, see Brinkgreve et al. (2008).

Hydraulic equations

The advective flow of the water phase is described via the generalized Darcy's law:

$$\mathbf{q}_l = -\frac{\mathbf{k}k_{rl}}{\mu_l}\left(\nabla P_l - \rho_l \mathbf{g}\right) \quad (14)$$

where μ_l is the dynamic viscosity of the pore liquid, g is the gravity acceleration, ρ_l is the liquid density. The tensor of intrinsic permeability $\mathbf{k}$ is defined by the Kozeny's model:

$$\mathbf{k} = \mathbf{k}_o \frac{\phi^3}{(1-\phi)^2}\frac{(1-\phi_o)^2}{\phi_o^3} \quad (15)$$

where ϕ is the porosity, ϕ_0 is a reference porosity, $\mathbf{k}_o$ is the intrinsic permeability for matrix with a porosity ϕ_0. The relative permeability k_{rl}, is derived from the Mualem-van Genuchten model:

$$k_{rl} = \sqrt{S_e}\left(1-\left(1-S_e^{1/\lambda}\right)^{\lambda}\right)^2 \quad (16)$$

where λ is a shape parameter for retention curve. The effective degree of saturation S_e is calculated as follows:

$$S_e = \frac{S_l - S_{rl}}{S_{ls} - S_{rl}} = \left(1+\left(\frac{P_g - P_l}{P_0}\right)^{\frac{1}{1-\lambda}}\right) \quad (17)$$

where S_{ls} and S_{rl} are the maximum and the residual degree of saturation, P_0 is a model parameter.

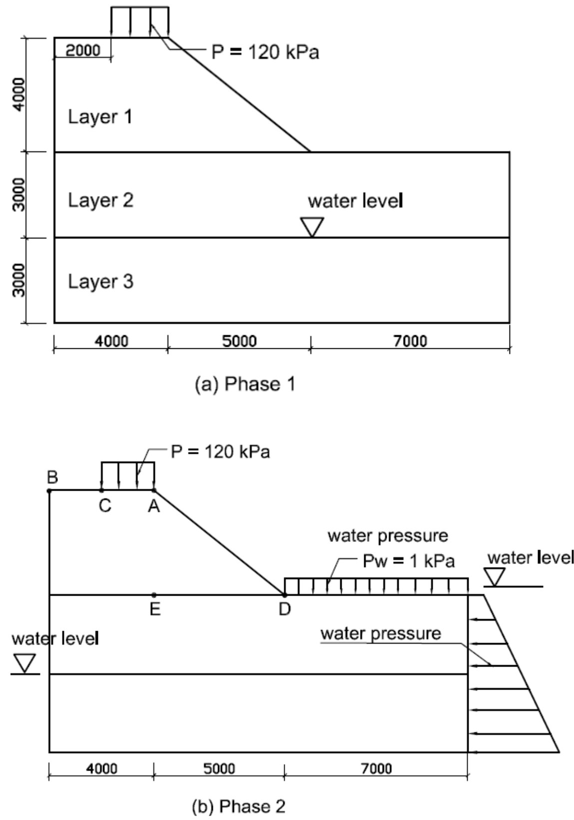


Figure 2 Scheme of the problem geometry for simulating the landslide process

MODEL PERFORMANCE IN A BOUNDARY VALUE PROBLEM

Description of the boundary value problem

The model is applied to simulate the landslide process in Ma River region. The reason for the instability phenomenon is assumed by the water level variation within the two seasons in Vietnam. In dry season water level is low and the river levee is in an unsaturated state. Because the soil has high strength at unsaturated state, the top levee can support an existing construction on it. When the rainy season comes, water level increases to the toe of the levee. With the infiltration process at the toe of the levee, the soil strength decreases and that may triggers off the instability phenomenon.

For numerical simulation of the landslide process, the geometry of the riverbank is simplified as it is given in Fig. The simulation process is divided into two phases. Phase 1 corresponds to the conditions during the dry season, when water level is low and the top edge of the riverbank is imposed to a distributed load $P = 120$ kPa. Water lever is taken to be 3.0 meters up from the bottom of the numerical model. Phase 2 corresponds to the conditions during the rainy season, when water level increases. It is assumed in this phase that the water level is 10 cm above the slope toe. The deformation and the suction redistribution is simulated using fully coupled HM model, namely BBM-VP. For comparison, we employ the DP-VP model where unsaturated flow is considered via the generalized Darcy's law but the coupling between suction and mechanical model parameters is not applied.

In CODE_BRIGHT initial water content is introduced by liquid pressure. Negative liquid pressure is used for unsaturated state. Positive liquid pressure is used for saturated soil. Gas pressure is assumed to be zero. In order to give water level is 3 meters from the bottom of the model, the water boundary pressure at bottom of the model P_l=0.03(MPa). Initial conditions for CODE_BRIGHT is presented in Table 1. Initial bulk soil density $\gamma_{initial}$ for Plaxis calculation is presented in Table 4.

Table 1 Initial conditions for CODE_BRIGHT

Parameter	Unit	Layer 1	Layer 2	Layer 3
ϕ_0	-	0.45	0.42	0.40
P_{l0}	MPa	-0.6	-0.05	0
e_0	-	0.818	0.724	0.667

Model Parameters

The properties of soil at the Ma riverbank are reported in Phạm et al. 2008. Because the dam was made of sandy clay, the mechanical parameters for BBM are taken to be the parameters obtained for a silty soil (Geiser et al. 1999) whose content and structure are close to that of the dam material. The mechanical parameters for BBM-VP are given in Table 2 and Table 3.

The corresponding parameters for DP-VP and MC models are presented in Table 4. The viscosity parameter is taken the same for both viscoplastic laws, i.e. Γ is equal to 250 s^{-1} for layers 1 and 2, and it is equal to 10 s^{-1} for layer 3, see Fig. 2. The other parameters are $N = 4$, and $F_o = 1.0$ MPa

Table 2 Parameters for the mechanical model

Parameter	Unit	Layer 1	Layer 2	Layer 3
α	-	0.3	0.3	0.3
γ	-	1	1	1
M_{dry}	-	1.33	1.33	1.4
M_{sat}	-	0.6	0.6	1.1
p_o^*	MPa	0.21	0.21	0.4
a	-	3	3	3

Table 3 Parameters for the mechanical model – continuation

Parameter	Unit	Layer 1	Layer 2	Layer 3
κ	-	0.007	0.007	0.007
$\lambda(0)$	-	0.032	0.032	0.032
r	-	0.2	0.2	0.2
β	-	0.01	0.01	0.01
p^c	MPa	0.05	0.05	0.1
k	-	0.01	0.01	0.01
e_o	-	0.818	0.724	0.667
E		50	50	120
ν	-	0.3	0.3	0.3

Table 4 Parameters for DP-VP and MC models

Parameter	Unit	Layer 1	Layer 2	Layer 3
φ	°	33	33	33
c	-	8.4	8.4	0.7
γ_{dry}	g/cm^3	14.6	14.6	15.4
γ_{wet}	g/cm^3	18.9	18.9	19.4
$\gamma_{initial}$	g/cm^3	15.9	15.9	18.5
E	kN/m^2	5000	5000	12000
ν	-	0.3	0.3	0.3

The water retention curve is expressed via the well known two-parameters van Genuchten model (van Genuchten, 1980). The parameters for the hydraulic constitutive equations are shown in Table 5.

Table 5 Hydraulic parameters assumed

Parameter	P_o	λ	k_o	ϕ_0
Unit	MPa	-	(m^2)	-
Layer 1	0.04	0.3	2.5×10^{-14}	0.3
Layer 2	0.04	0.3	2.5×10^{-14}	0.3
Layer 3	0.04	0.3	1.5×10^{-14}	0.3

RESULTS AND DISCUSSION

Result of numerical simulation using coupled HM model

The line from point A to point E is selected for observation of displacements and the degree of saturation (Fig). In the first phase, the displacements at point A are relatively small as compared to ones at points B and C. In the second phase, when the water level increases to the toe of the levee the degree of saturation increases and the suction decreases (Figure 2 and Figure 3). The distribution of the degree of saturation within the levee body is shown in Figure 4a. Consequently, the soil strength at the levee toe decreases. The decrease of soil strength and the volumetric softening behaviour are expressed by Eq. (8) and Eq. (10). It is visible that the settlement at point A increases rapidly as compared to the one at point B (Figure 3). The difference in the displacements at point A and point B may induce a crack and may trigger off the landslide. In reality, a crack most possibly will develop at the weakest locations, e.g. a place with imperfection in levee body or the place with maximum strain deviation. Assuming that the levee body is built of homogeneous materials, the slide surface can be determined owing to localization of plastic deviatoric strain. With this method, the slide surface can be identified as it is shown in Figure 4b.

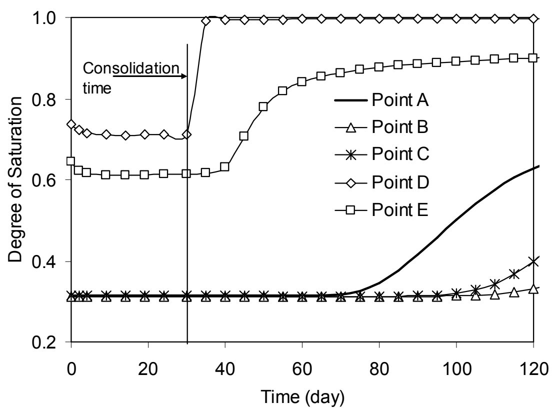


Figure 2 Degree of saturation vs. time

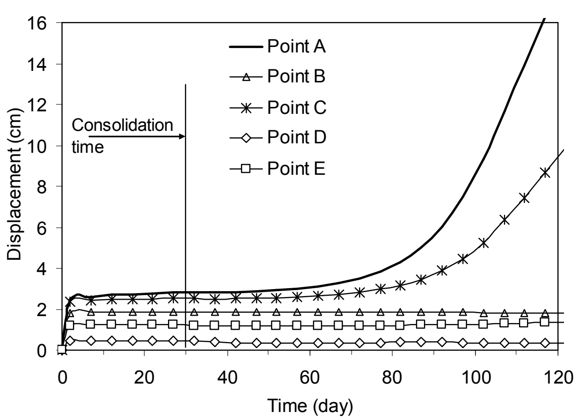
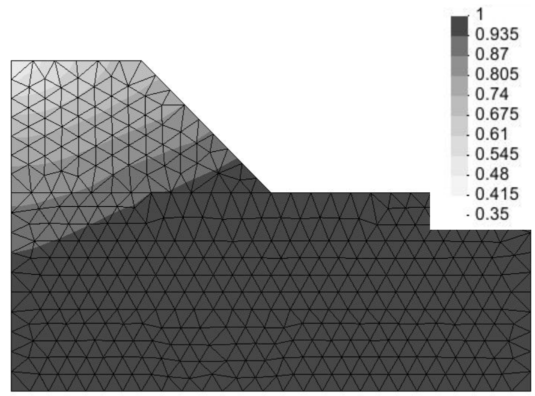


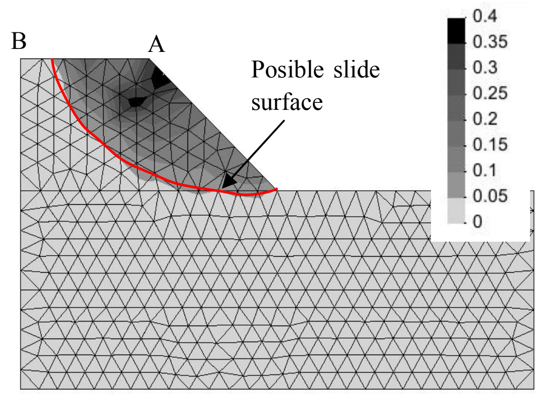
Figure 3 Displacements vs time

Verification against other models and FE-code

For assessing the efficiency of the fully coupled hydro-mechanical (HM) model the results obtained using BBM-VP model are compared with the results of the simulation of the process employing other partly coupled or uncoupled models. Two models are considered for this purpose. Both models have as a background the Mohr-Coulomb failure criterion. In the first case, the simulation of the landslide performance was done using linear elastic-viscoplastic model based on Drücker-Prager failure yield criterion (DP-VP). The DP-VP model as it is implemented in the FE code CODE_BRIGHT takes into account the unsaturated flow, but there is no coupling between suction and the soil stiffness. The second case uses as a constitutive law the Mohr-Coulomb model as it is implemented in the FE code Plaxis (Brinkgreve et al., 2008). The hydraulic behaviour is considered through a saturated flow via the Darcy's law and a prescribed gravitational water level.



(a)



(b)

Figure 4 (a) Distribution of the degree of saturation on the 150th day; (b) Plastic deviatoric strain on the 150th day - BBM-VP

Figure 5 presents the evolution of settlement at point A (at the edge of the dike top) and point B (far from the dike edge). In the first phase, the settlement at point A calculated with the coupled HM and BBM-VP model and the settlement obtained via uncouple HM simulation with Mohr-Coulomb model develop in similar manner. The final settlement at point A in the first phase calculated via Mohr-Coulomb model without suction is nearly linear with time, because no viscosity equation is used. In the second phase, because of reduction of the stiffness and shear strength, the settlement at point A calculated via BBM-VP increases rapidly. Contrary, the settlement at point A obtained in the simulation using the Mohr-Coulomb model changes insignificantly (Figure 5a). Table 6 summarizes the final (on the 150th day) difference in the settlement obtained by using the three different models. During the first phase at point B, the process of the development of the settlement is similar to that at point A during the first phase, but the magnitude of settlement is relatively smaller at point B. In the second phase, the settlement at point B obtained changes insignificantly and it is valid for all models. The settlement at point B takes negative values. This phenomenon can be explained by an increase of the water pressure on slope toe of the levee, which uplifts the levee body on the left of the calculated model.

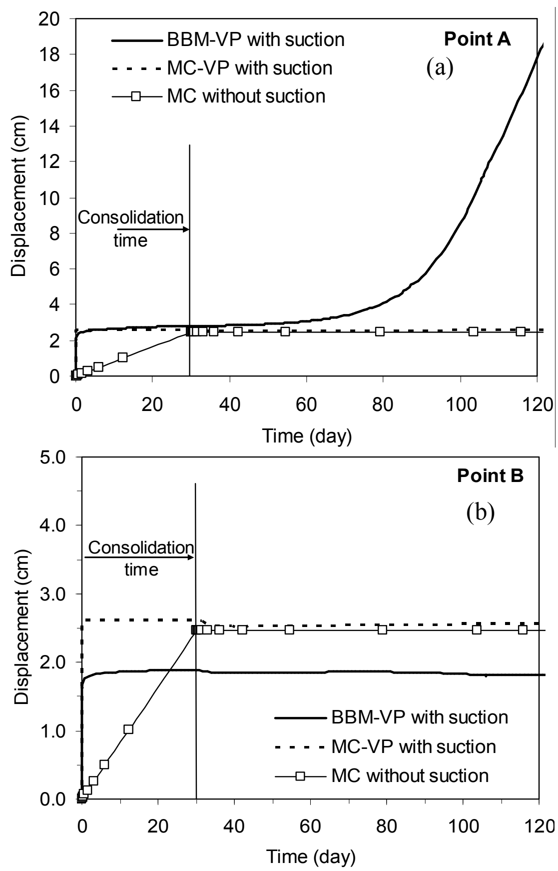


Figure 5 Displacements vs. time: comparison of different model results. (a) – at point A, (b) – at point B

Table 6 Total difference in displacement between points on the 150th day

Model	Point A-B (cm)	Point C-B (cm)
BBM-VP with suction	40.63	17.57
DP-VP with suction	0.13	0.49
MC without suction	1.07	1.26

CONCLUSION

Landslide process that may be applicable for the conditions at Ma River has been numerically simulated using different mathematical models. An elasto-viscoplastic model for unsaturated soil based on the BBM model was applied for simulating the deformation process due to the water level change and the influence of suction. The obtained results are compared with the results of simulation of the same process but using different constitutive models and different finite element codes. It is demonstrated the dependence of the settlement on the utilized interpretation of the hydro-mechanical process and the influence of the suction as well as the time-dependence on the final difference in displacements at the levee top. This way we assess the failure potential of the levee based on the stresses and deformations arising from the levee top footing and inter-annual climate oscillations. As a conclusion, it may be recommended a detailed and critical assessment of the constitutive model features before using the calculated stress and strain fields to predict the stability of a riverbank. Especially, it has to be pointed that the omission of the effect of suction from slope stability calculations may yield wrong predictions.

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