ANOTHER METHOD OF LOGIC SYNTHESIS OF DIGITAL COUNTING CIRCUITS

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Abstract. In order to synthesize automat (in this case digital counters), the minimizing internal states is of particular significance and plays a decisive role in the results of synthetic circuit. This can be done in many ways, but the use of Karnaugh map is considered optimal. However, this process has some disadvantages that it can not be overcome when the number of input variants is large. In experience, if the number of variants is 7, manual minimization of circuit functions using Karnaugh map arises many difficulties and even become impossible if over 10 variants are available.

In order to deal with this weakness, it is both necessary and rational to use computer in logical synthesis of counting circuit. This is the aim of this article.

1. Synthesizing counting circuits using similar forms

For the method of synthesizing digital counting circuits using computers is firstly primarily based on the results achieved through the Karnaugh map [1]. By there results drawning the general laws of circuit functions for each synchronous and asynchronous counters, for each Flip-Flop (FF) and for each kind of codes. *These general laws* help to develop *mathematical models* as well as *computer programmes* which enable the fastest definition of minimized circuit functions of each desired counters.

Let us investigate, for example, the input states R_2, S_2 and outputs states Q_2 of RS – FF in synchronous counters, real binary, 4 inputs (k = 4). The input states R_2, S_2 as well as outputs states Q_2 are given in table 1.

Table 1. The input states R_2, S_2. ε is counting state, $\varepsilon = 2^k - 1 = 2^4 - 1 = 15 = m - 1$, with m is a cardinal number Q_2 is an output state corresponding inputs states S_2 and R_2

ε	Q_2	R_2	S_2	ε	Q_2	R_2	S_2	ε	Q_2	R_2	S_2	ε	Q_2	R_2	S_2
0	0	d	0	4	0	d	0	8	0	d	0	12	0	d	0
1	0	0	1	5	0	0	1	9	0	0	1	13	0	0	1
2	1	0	d	6	1	0	d	10	1	0	d	14	1	0	d
3	1	1	0	7	1	1	0	11	1	1	0	15	1	1	0

From table 1, we can form impulse diagrams for both Q_2, R_2 and S_2 (figure 1)

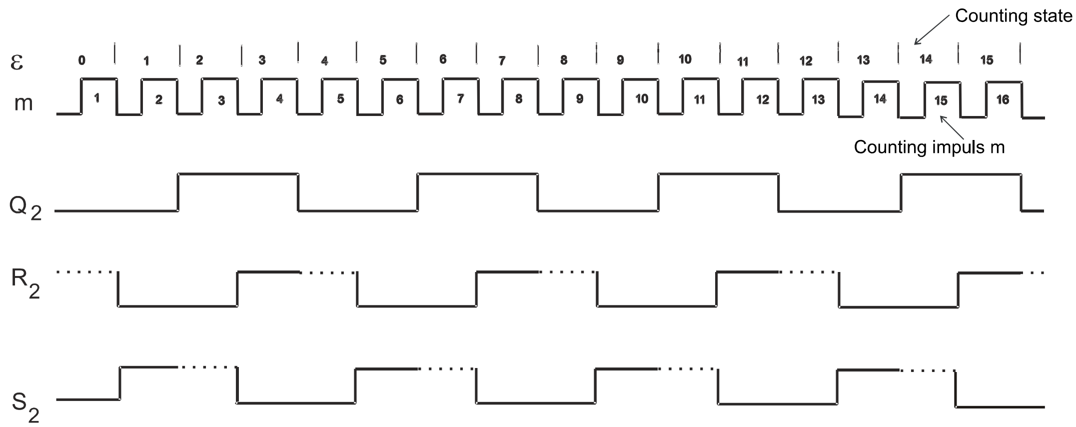


Figure 1: Impulse diagram of Q_2, R_2, and S_2. The dots represent non-defined state 'don't care', which receive either value 1 or 0 when circuit function is reduced and which can be used or not be depending on certain cases

In this case we particularly study the input state R_2.

If in minimizing circuit functions using Karnaugh map state 'don't care' is given value 0, we can have an impulse diagram of R_2 as in figure 2.

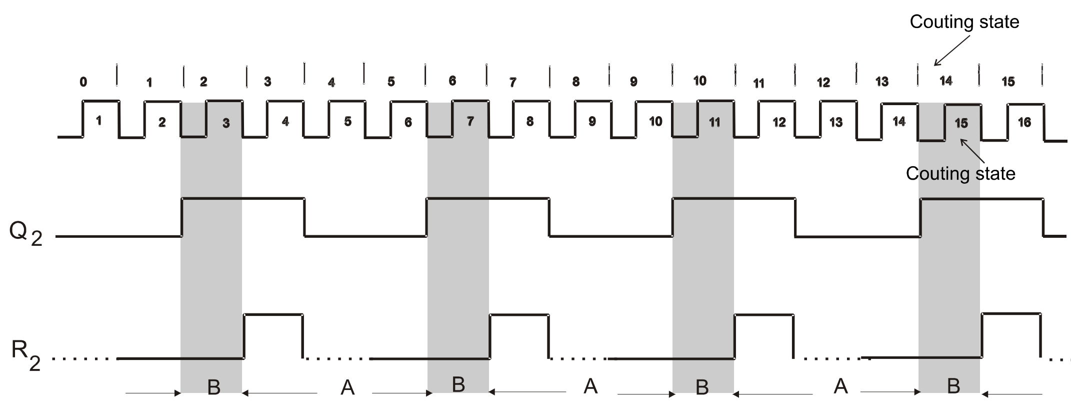


Figure 2: Impulse diagram of Q_2 and R_2 to describe forms of circuit functions of R_2

From figure 2 we can see that if Q_2 and R_2 impulses are the same state, the responding circuit functions have the same form, here called form 1. If Q_2 impulse flank positive but R_2 impulse flank negative, then circuit functions have the same form 2. Figure 3 describes the appearance of form 1 and form 2 on ε axis.

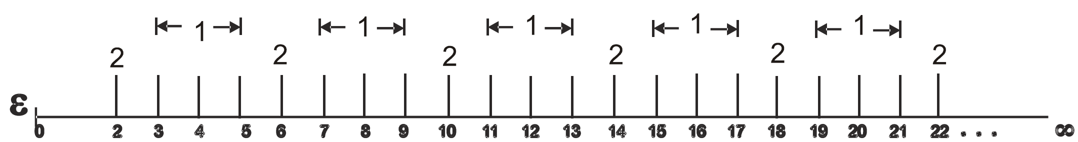


Figure 3: Description of form 1 and form 2 on ε axis

From figure 3 we can see that if ε = 2, 6, 10, 14…, the corresponding impulse diagrams have the same for (form 1) and if ε = (3, 4, 5), (7, 8, 9)… the corresponding impulse diagrams have the same form (form 2). Therefore we say impulse diagrams corresponding ε in a definite period are similar.

The concept of 'similar' is understood in the following way: Two similar circuit functions are two functions that have the same mathematical structure, such as the same "sum of products" or "product of sums", or similar sum, similar product, but the connotations are different.

This concept of 'similar' is the basis on which mathematical models for circuit functions of counters, in this case synchronous RS-FF counter, real binary code, are formed.

Also from table 1, using Karnaugh map method, we can define circuit functions corresponding to inputs R_2 (Table 2).

Table 2: circuit functions corresponding to input R_2 of RS - FF - Counter, k = 4, ε = 0 to 15

ε	R_2	ε	R_2	ε	R_2	ε	R_2
2	Q_2	**6**	$Q_1Q_2+ Q_2Q_3$	**10**	$Q_1Q_2+ Q_2Q_4$	**14**	$Q_1Q_2+Q_2Q_3Q_4$
3 4 5	Q_1Q_2 Q_1Q_2 Q_1Q_2	7 8 9	Q_1Q_2 Q_1Q_2 Q_1Q_2	11 12 13	Q_1Q_2 Q_1Q_2 Q_1Q_2	15	Q_1Q_2

From figure 2, figure 3, table 2 and the concept of "similar" we can see that circuit functions have two main forms:

Form 1:

$$R^{(1)}_{l,\varepsilon} = Q_1.Q_2....Q_l = \prod_{i=1}^{\ell} Q_i = A \qquad \forall\varepsilon \in \{2,6,10,14\} \tag{1}$$

Form 2:

$$R^{(2)}_{l,\varepsilon} = A + \prod_{i=1}^{t} Q_i = A + B\,,\ \ \forall\varepsilon \notin \{2,6,10,14\} \tag{2}$$

with

$$B = \prod_{i=1}^{t} Q_i \tag{3}$$

If we can prove that the existence of form 1 and 2 follows a certain law, for example form 1 and 2 exist at the same time in a repetitive period with denominator $\Delta\varepsilon = 2^{\ell} = 4$ (in this case $\ell = 2$): ε_1 = {(3, 4, 5), (7, 8, 9), (11, 12, 13)…}; ε_2 = {2, 6, 10, 14…} and if we can *define the circuit functions* of form 1 (as well as form 2) with ε_1 = (3, 4, 5) (as well as ε_2 = 2), we can define all circuit functions of input R_2 (as well as S_2) with any ε counting state.

1.1 Prove the existence of circuit functions follows a certain law

The problem is we have to prove with any unlimited great k variable input, that is with any unlimited great ε counting state, the impulse diagram always follows a certain law, that is always exist form 1 and 2 corresponding a repetitive period with $\Delta\varepsilon = 2\ell$ (in the case of counter RS=FF).

Assuming that $f(\varepsilon) = R^{(1)}_{1,\varepsilon}$ is a function satisfying term Dirichlet of Fourier theorem on period [3, 4, 5] = [a,b]. In order to develop $f(\varepsilon)$ into Fourier series, we form a periodic function $g(\varepsilon)$ having a period either bigger or equal to (b – a) so that $g(\varepsilon) = f(\varepsilon), \forall\varepsilon \in [a,b]$

Obviously there are many ways to develop $g(\varepsilon)$ into Fourier series. For each $g(\varepsilon)$ there is a corresponding Fourier series, therefore there are a number of Fourier series demonstrating $f(\varepsilon) = R_{1,\varepsilon}^{(1)}$. Similarly, $f(\varepsilon) = R_{1,\varepsilon}^{(2)}$ can also be developed into a Fourier series. To put it simple, the circuit function $f(\varepsilon) = R_{\ell,\varepsilon}^{(1)} + R_{\ell,\varepsilon}^{(2)}$ with every ε is periodicall with period $\Delta\varepsilon = 2^{\ell}$ (in this case $\ell = 2 \rightarrow \Delta\varepsilon = 4$) (Figure 4).

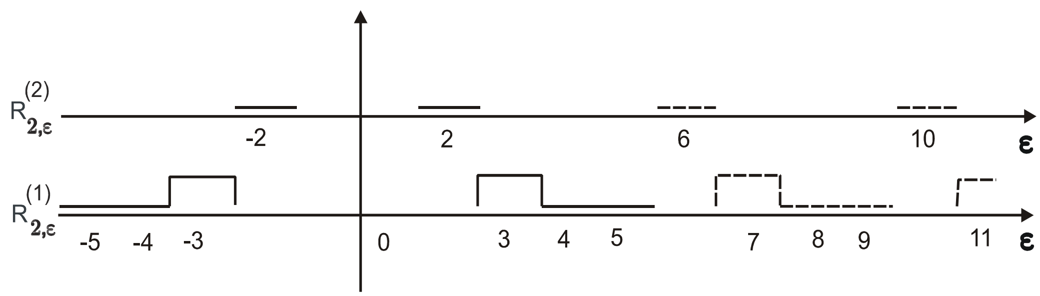


Figure 4: Circuit function developed into Fourier series with $\Delta\varepsilon = 4$

Now that we can assert that with any variable input ℓ, that is the counter can (theoretically) count to infinite number, then the impulse diagram of circuit function *change periodically in those periods which have similar impulses*, that is circuit functions always have form 1 and 2 according to certain $\Delta\varepsilon$.

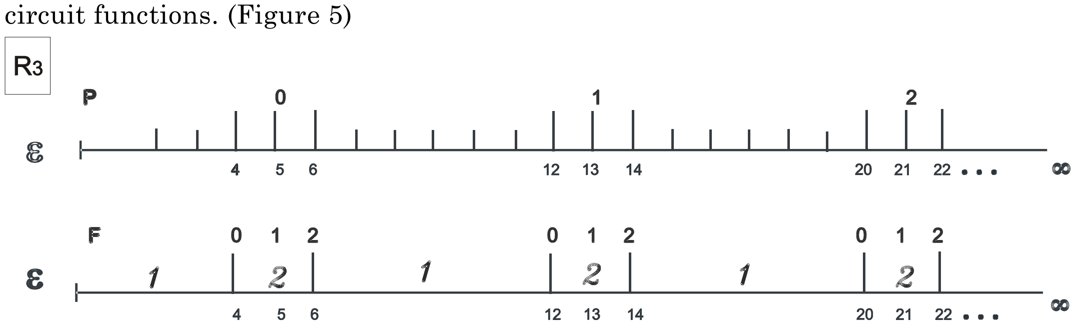
We particularly study the characters of this law for counters, firstly the circuit function in form 2.

In order to identify and analyze the forming of form 2 of input state $R_{\ell,\varepsilon,}$ we investigate the circuit function of for example R_3 in RS - FF - Counter with k = 6 (table 3).

Table 3. Circuit function with input state $R_{3,\varepsilon}$ of RS - FF - Counter with k = 6, ε = 0 to 64, in which: P is the periodical existence of circuit function form 2; F is the frequency of existence of circuit function in each period P, with corresponding denominator $\Delta\varepsilon = 2^{\ell}$

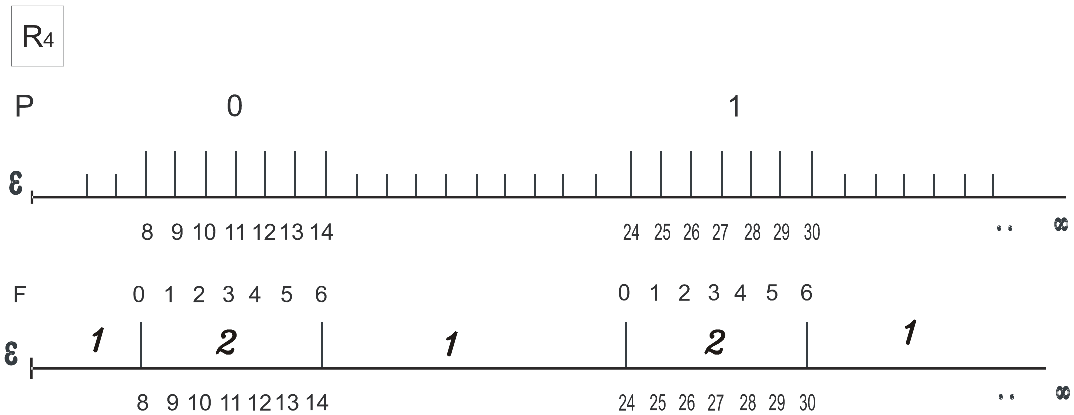
P	F	ε	*Circuit function in form 2 of R_3*	ε_3
	0	**4**	Q_3	$2^2 + \mathbf{0}.2^3 + 0$
0	1	5	$Q_1\ Q_3$	$2^2 + \mathbf{0}.2^3 + 1$
	2	6	$Q_2\ Q_3$	$2^2 + \mathbf{0}.2^3 + 2$
1	0	**12**	$Q_1\ Q_2\ Q_3 + \quad Q_3Q_4$	$2^2 + \mathbf{1}.2^3 + 0$
	1	13	$Q_1\ Q_2\ Q_3 + \quad Q_1Q_3Q_4$	$2^2 + \mathbf{1}.2^3 + 1$
	2	14	$Q_1\ Q_2\ Q_3 + \quad Q_2Q_3Q_4$	$2^2 + \mathbf{1}.2^3 + 2$
2	0	**20**	$Q_1\ Q_2\ Q_3 + \quad Q_3Q_5$	$2^2 + \mathbf{2}.2^3 + 0$
	1	21	$Q_1\ Q_2\ Q_3 + \quad Q_1Q_3Q_5$	$2^2 + \mathbf{2}.2^3 + 1$
	2	22	$Q_1\ Q_2\ Q_3 + \quad Q_2Q_3Q_5$	$2^2 + \mathbf{2}.2^3 + 2$
3	0	**28**	$Q_1\ Q_2\ Q_3 + \quad Q_3\ Q_4Q_5$	$2^2 + \mathbf{3}.2^3 + 0$
	1	29	$Q_1\ Q_2\ Q_3 + \quad Q_1Q_3\ Q_4Q_5$	$2^2 + \mathbf{3}.2^3 + 1$
	2	30	$Q_1\ Q_2\ Q_3 + \quad Q_2Q_3\ Q_4Q_5$	$2^2 + \mathbf{3}.2^3 + 2$
4	0	**36**	$Q_1\ Q_2\ Q_3 + \quad Q_3Q_6$	$2^2 + 4.2^3 + 0$
	1	37	$Q_1\ Q_2\ Q_3 + \quad Q_1Q_3Q_6$	$2^2 + \mathbf{4}.2^3 + 1$
	2	38	$Q_1\ Q_2\ Q_3 + \quad Q_2Q_3Q_6$	$2^2 + \mathbf{4}.2^3 + 2$
5	0	**44**	$Q_1\ Q_2\ Q_3 + \quad Q_3\ Q_4Q_6$	$2^2 + \mathbf{5}.2^3 + 0$
	1	45	$Q_1\ Q_2\ Q_3 + \quad Q_1Q_3\ Q_4Q_6$	$2^2 + \mathbf{5}.2^3 + 1$
	2	46	$Q_1\ Q_2\ Q_3 + \quad Q_2Q_3\ Q_4Q_6$	$2^2 + \mathbf{5}.2^3 + 2$
.			.	
.			.	
.			.	

From table 3 we can form the relation between P, F and ε for two forms of circuit functions. (Figure 5)



with $\Delta\varepsilon = 2^{\ell} = 2^3 = 8$

Similarly, with $R_{4:}$



with $\Delta\varepsilon = 2^{\ell} = 2^4 = 16$

Figure 5: Description of the relation between P, F of circuit function R_3 and R_4 on ε axis

Circuit functions of form 1 and 2 depend on ε, ℓ ($\ell \in$ k). This dependence is demonstrated by the periodical existence of P and frequency of existence in each period F (See table 3 and Figure 4)

From table 3 and figure 5 we can identify the relation between ℓ, ε, P and F, that is the relation as well as the mathematical models showing the existence law of circuit functions:

$$\varepsilon_{\ell} = 2^{\ell-1} + P.2^{\ell} + F \tag{4}$$

From (4) we have:

$$\frac{\varepsilon_l}{2^l} = P + \frac{2^{l-1}+F}{2^l} \tag{5}$$

Put

$$I = I = 2^{l-1} + F \tag{6}$$

Apply to (6) we have

$$P = \frac{\varepsilon - I}{2^l} \tag{7}$$

From (5), (6), and (7) we see that parameter I can show the complete frequency and periodical existence of circuit function.

Call E set of I from I_0 to I_{max}, we have:

$$E = \{I_0, I_1, I_2, \ldots, I_{max}\} \tag{8}$$

$$E = \{F_0 + 2^{\ell-1}, F_1 + 2^{\ell-1}, \ldots, F_{max} + 2^{\ell-1}\} \tag{9}$$

or
$$E = \{\, 2\ell\text{-}1,\ 1 + 2\ell\text{-}1,\ 2 + 2\ell\text{-}1\ \ldots,\ 2\ell - 2\} \tag{10}$$

with $F_{max} = 2^{\ell} - 2$

Obviously now the value of set E shows the complete parameters of periodical as well as frequency existence of forms of circuit functions (from F_0 to F_{max}), in other words, in order to identify P and F we only have to identify the value of set E. Figure 6 shows the existence of circuit function with E corresponding R_3 ($\ell = 3$).

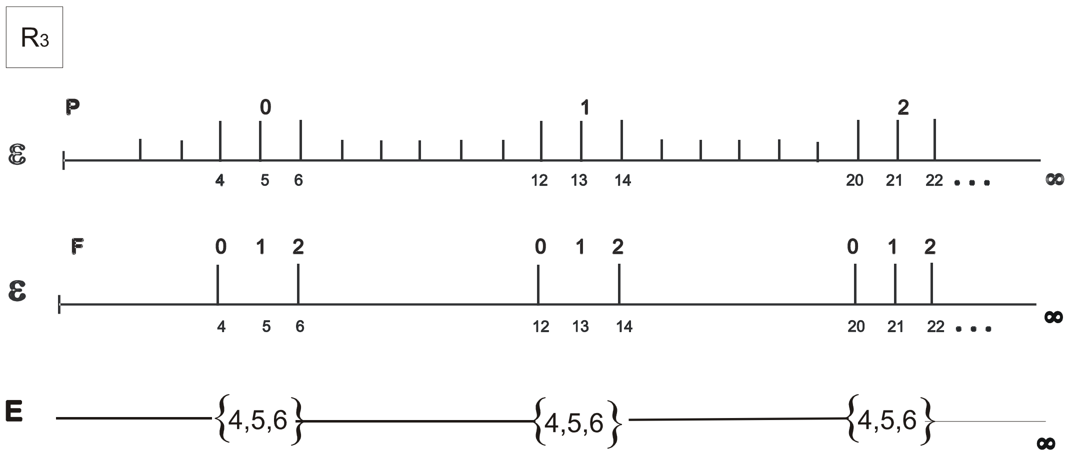


Figure 6: Describes the existence of period P and frequency F through set E

If we look into figure 6, we can see that for each R_3, if $E \in \{4,5,6\}$ the circuit function will have form 2 and $E \notin \{4,5,6\}$ form 1.

1.2 Identifying circuit function

Another problem is that we have to identify the circuit functions of input ℓ for any counting state ε, in other words, we have to identify to relation between input state R_ℓ and output state Q_ℓ through counting state ε.

Identify circuit function form 1:

With $A = \prod_{i=1}^{\ell} Q_i$, we can obviously identify circuit function for any $\ell \in$ k. for example the third input of RS-FF - counter, that is $\ell = 3$, has circuit equation as following:

$$R_{3,\,\varepsilon} = \prod_{i=1}^{3} Q_i = Q_1 \,.\, Q_2 \,.\, Q_3 \qquad \varepsilon \text{ satisfies } E \notin \{4,5,6\}$$

with any ℓ we have:

$$R_{l,\varepsilon} = \prod_{i=1}^{l} Q_i = Q_1.Q_2...Q_l \qquad \varepsilon \text{ satisfies } E \notin \{2^{\ell-1},\ 1 + 2^{\ell-1},\ 2 + 2^{\ell-1}\ \ldots,\ 2^{\ell} - 2\}$$

** Identifying circuit function form 2:*

The problem is to identify circuit functions $B = \prod_{i=1}^{k} Q_i$

Through investigating circuit functions forms B for input R (as well as S) of counter RS – FF we have an interesting remark: for each different ε counting state there is very different circuit function, apparently not following any law (see table 3) but if we look at the relation between binary number and decimal number we will see that the binary number showing the total weight η_B of Q_ℓ equals to the decimal number showing counting state ε, that is, η_B = ε.

Assuming that the counter has k inputs, then the corresponding weight to each input will be:

Q_k	......	Q_ℓ	......	Q_5	Q_4	Q_3	Q_2	Q_1
η_k	......	η_ℓ	......	η_5	η_4	η_3	η_2	η_1
2^{k-1}	......	$2^{\ell-1}$	......	2^4	2^3	2^2	2^1	2^0

Now the total weight of Q_1 to Q_k is

$$\eta_B = b_{k+1} \,.\, 2^k + b_k \,.\, 2^{k-1} + \ldots.. + b2_2 + b^0{}_1 = \sum_{i=1}^{k} b_i.2^{i-1} \tag{11}$$

And from the conclusion ε = 19, k = 5 we have

$\eta_B = 19 = b_6 \,.\, 2^5 + b_5 \,.\, 2^4 + b_4 \,.\, 2^3 + b_3 \,.\, 2^2 + b_2 \,.\, 2^1 + b_1 \,.\, 2^0$

$= b_6 \,.\, 32 + b_5 \,.\, 16 + b_4 \,.\, 8 + b_3 \,.\, 4 + b_2 \,.\, 2 + b_1 \,.\, 1$

$= 0.32 + 1.16 + 0.8 + 0.4 + 1.2 + 1.1$

That factors b_6, b_4, b_3 must equal to 0 and $b_5 = b_2 = b_1 = 1$ when $\eta_B = 19$

In other words, when k = 5 with ε = 19 the circuit function B will have Q_ℓ = (Q_4, Q_1. Q_0), that is, $B_{19,5} = \prod_{i=1}^{5} Q_i = Q_0.Q_1.Q_4$

Now we can form a formula to find B:

$$B = \prod_{i=1}^{k} \left(Q_i + \overline{b_i}\right), \text{ with } \overline{b_i} = \begin{cases} 1 & b_i = 0 \\ 0 & b_i = 1 \end{cases} \tag{12}$$

From this we can form the formula to identify the circuit function for each corresponding $R_{l,\varepsilon}$ of RS - FF - Counter

$$R_{\ell,\varepsilon} = \Delta_A.\prod_{i=1}^{\ell} Q_i + \Delta_B.Q_\ell \prod_{i=1}^{t} (Q_i + \overline{b_i}).\overline{\Delta}\ell \tag{13}$$

In which:

$$\Delta_A = \begin{cases} 1 & \varepsilon \geq 2^\ell - 1 \\ 0 & \text{other cases} \end{cases}$$

Δ_A shows the condition for the existence of form A

$$\Delta_B = \begin{cases} 1 & I = F + 2^{l-1} \in E = \left\{2^{l-1},\ 1+2^{l-1}, ...,\ 2^{l}-2\right\} \\ & \eta_B = \varepsilon - 2^{\ell - 1} = \sum_{i=1}^{t} b_i.2^{i-1} \quad t = \left[ld\eta_B + 1 \right] \\ 0 & \text{other cases} \end{cases}$$

Δ_B shows the condition for the existence of form B

$$\Delta_\ell = \begin{cases} 1 & \ell = 1 \\ 0 & \text{other cases} \end{cases}$$

Δ_ℓ shows the condition for the existence of input $\ell = 1$

The identification of $S_{l,\varepsilon}$ is done similarly.

2. Conclusion

From the above results we can form a mathematical model for RS - FF - Counter and through their relation [1] (figure 7) we can identify the circuit fuction, that is the mathematical model for all counters namely JK-FF, T-FF, D-FF…

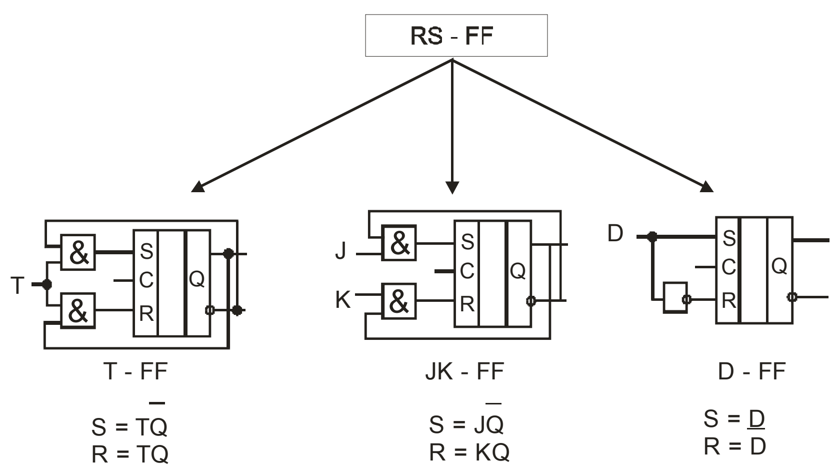


Figure 7: The relation between RS-FF with T-, JK-, D-FF

For example define the circuit function of D – FF. From figure 7 we can see that circuit function of D – FF is:

$$D = S$$

or

$$D = \overline{R}$$

That is:

$$D_{\ell,\eta} = \overline{R}_{\ell,\varepsilon} = \overline{\Delta_A.\prod_{i=1}^{\ell} Q_i + \Delta_B.Q_{\hbar}\prod_{i=1}^{t}(Q_i + \overline{b}_i).\overline{\nabla}_{\ell}} \qquad (14)$$

With these mathematical models and with software such as Pascal, C^{++}..., and especially Mathlab we can easily designsynchronous or asynchronous counters, for all codes, with any cardinal numbers using computers.

References

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