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Quantum kinetic equation in the quantum hadrondynamics (QHD-I) model

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Abstract. Within the framework of the Walecka model (QHD-I)[1] the renormalized effective Dirac equation and the kinetic equation for fermion are presented. In fact, the fermion propagator in the medium is dramatically different from that in the vacuum. The main feature is the treating of the fermion distribution in non equilibrium, which depends on the interaction rate involving temperature.

*Keyworks:*field Theory, nuclear Theory.

1. Introduction

At present the theoretical study of quantum fields at finite temperature and density turns out to be more and more important for description of wide variety of physical effects in medium: in condensed matter [2,3], in stellar astrophysics [3,4], in a QED or QCD plasma [5,6]. The effective real-time Dirac equation in medium and the kinetic equation not only may provide an approximation beyond two-loop calculations, but also can be treated the correlation effects- those that are extremely important for physical processes near equilibrium [8].

In this paper we focus on the QHD-I model of non-zero density. We investigate in detail the fermion propagator and its relaxation and thermalization through the interaction with scalar and neutral vector mesons in the matter. Furthermore, the kinetic equation for fermion in the real time is shown the relation between the fermion distribution and the interaction rate.

The paper is organized as follows. In Sec. II the QHD-I and the real time formalism are presented. Sec. III is devoted to considering the renormalized effective Dirac equation. In Sec. IV we carry out the quantum kinetic equation for fermion in the QHD-I. The discussion and conclusion are given in Sec. V.

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2. Formalism

2.1. The Quantum Hadrondynamics (QHD-I)

We start with the Lagrangian density

$$\begin{aligned} L_o =& \bar{\mathbf{\Psi}} \left(i\gamma^\mu \partial_\mu - M_0 - g\mathbf{\Phi} - g_\omega \gamma^\mu \mathbf{V}_\mu \right) \mathbf{\Psi} \\ &+ \frac{1}{2} \left(\partial^\mu \mathbf{\Phi} \partial_\mu \mathbf{\Phi} - m_0^2 \mathbf{\Phi} \right) - \frac{\lambda^2}{4} \mathbf{\Phi}^4 + \frac{1}{2} m_W^2 \mathbf{V}^\mu \mathbf{V}_\mu - \frac{1}{4} \mathbf{F}^{\mu\nu} \mathbf{F}_{\mu\nu}, \end{aligned} \tag{1}$$

where $\mathbf{\Psi}, \mathbf{\Phi}$ and $\mathbf{V}_\mu$ are the field operators of fermion, scalar and vector meson, respectively, and $\mathbf{F}^{\mu\nu} = \partial^\mu \mathbf{V}^\nu - \partial^\nu \mathbf{V}^\mu$.

In the medium of finite density (the nuclear matter), the symmetry of the ground state $|F\rangle$ yields

$$\begin{aligned} \left\langle F \left| \bar{\mathbf{\Psi}} \right| F \right\rangle = 0; \qquad & \langle F | \mathbf{\Psi} | F \rangle = 0, \\ \langle F | \mathbf{\Phi} | F \rangle = v; \qquad & \langle F | \mathbf{V}_\mu | F \rangle = \delta_{0\mu} \omega_\mu, \end{aligned} \tag{2}$$

where v and ω are the independent of space - time coordinate owing to the homogeneity of nuclear matter.

Adding to (2.1) a term, for example

$$L_0 \to L = L_0 + c\mathbf{\Phi},$$

which leads to an explicit chiral symmetry breaking.

By shifting the scalar and vector fields $\mathbf{\Phi}$ and $\mathbf{V}_\mu$ respectively

$$\mathbf{\Phi} \Rightarrow v + \mathbf{\Phi}; \quad \mathbf{V}_\mu \Rightarrow \delta_{0\mu} \omega_\mu + \mathbf{W}_\mu, \tag{3}$$

the Lagrangian density (1) in the presence of external sources now takes the form

$$\begin{aligned} \mathfrak{L} =& \bar{\mathbf{\Psi}} \left(i\gamma^\mu \partial_\mu - M_N - g\mathbf{\Phi} - g_\omega \gamma^\mu \mathbf{W}_\mu \right) \mathbf{\Psi} \\ &+ \frac{1}{2} \left[(\partial_\mu \mathbf{\Phi})^2 - m^2 \mathbf{\Phi}^2 \right] - \lambda^2 v \mathbf{\Phi} (\mathbf{\Phi}^2 + v^2) - \frac{\lambda^2}{4} (\mathbf{\Phi}^4 + v^4) \\ &+ \frac{1}{2} m_W^2 \mathbf{W}^\mu \mathbf{W}_\mu - g_\omega \bar{\mathbf{\Psi}} \gamma^\mu \delta_{0\mu} \omega_\mu \mathbf{\Psi} - \frac{1}{4} \mathbf{F}^{\mu\nu} \mathbf{F}_{\mu\nu} \\ &+ \bar{\eta} \mathbf{\Psi} + \bar{\mathbf{\Psi}} \eta + J_\phi \mathbf{\Phi} + J^\mu \mathbf{W}_\mu + c(v + \mathbf{\Phi}), \end{aligned} \tag{4}$$

where

$$M_N = M_0 + gv; \qquad m_\phi^2 = m_0^2 + 3\lambda^2 v^2 \tag{5}$$

are the masses of nucleons and scalar meson in the medium. The QHD - I is a renormalizable model. In the exact chiral limit, the parameter $c = 0$.

Now, we write the bare fields and sources in terms of the renormalized quantities (referred to a subscript r by introducing the wave function renormalization constants Z_ψ, Z_ϕ, Z_ω, the vertex renormalization $Z_g, Z_{g\omega}, Z_\lambda$ and the mass counter-terms $\delta M, \delta m, \delta m_W$) as follows

$$\begin{aligned} \mathbf{\Psi} =& Z_\psi^{1/2} \mathbf{\Psi}_r, \qquad & \mathbf{\Phi} = Z_\phi^{1/2} \mathbf{\Phi}_r, \\ \bar{\mathbf{\Psi}} =& Z_\psi^{1/2} \bar{\mathbf{\Psi}}_r, \qquad & (\mathbf{W}^\mu, \omega^\mu) = Z_\omega^{1/2} (\mathbf{W}_r^\mu, \omega_r^\mu), \end{aligned} \tag{6}$$

the external sources

$$\begin{aligned} \eta =& Z_\psi^{-1/2}\eta_r, \qquad J_\phi = Z_\phi^{-1/2}J_r, \\ \bar{\eta} =& Z_\psi^{-1/2}\bar{\eta}_r, \qquad L^\mu = Z_\omega^{-1/2}J_r^\mu, \end{aligned} \tag{7}$$

the coupling constants

$$\begin{aligned} g_r =& gZ_\phi^{1/2}Z_\psi/Z_g, \qquad g_\omega^r = g_\omega Z_{g_\omega}^{1/2}, \\ \lambda_r^2 =& \lambda^2 Z_\phi^2/Z_\lambda, \qquad c_r = cZ_\phi^{1/2}, \end{aligned} \tag{8}$$

and the masses

$$M_N = (M+\delta M)/Z_\psi, m_\phi^2 = (m^2+\delta m^2)/Z_\phi, m_\omega^2 = (m_W^2+\delta m_W^2)/Z_\omega, \tag{9}$$

where M, m^2 and m_ω^2 are the renormalized masses.

With the above definitions, the Lagrangian (4) can be rewritten as (we have suppressed the subscript r for notational simplicity)

$$\mathfrak{L}_{QHD-I} = \mathfrak{L}_{MF} + \mathfrak{L}_L + \mathfrak{L}_{source} + \mathfrak{L}_{SB}, \tag{10}$$

where

$$\mathfrak{L}_{MF} = -\frac{1}{4}\lambda^2 Z_\lambda v^2 + \frac{m_W^2+\delta m_W^2}{2}\mathbf{W}^\mu \mathbf{W}_\mu \tag{11}$$

$$\begin{aligned} \mathfrak{L}_L =& \bar{\mathbf{\Psi}}\left[iZ_\psi\gamma^\mu\partial_\mu - (M+\delta_\mu) - gZ_g\mathbf{\Phi}\right]\mathbf{\Psi} \\ &- g_\omega Z_{g_\omega}\bar{\mathbf{\Psi}}\gamma^\mu(\omega_\mu + \mathbf{W}_\mu)\mathbf{\Psi} - \frac{m_W^2+\delta m_W^2}{2}(\omega^\mu\omega_\mu + 2\mathbf{W}^\mu\mathbf{W}_\mu) \\ &+ \frac{1}{2}\left[Z_\phi(\partial_\mu\mathbf{\Phi})^2 - (m^2+\delta m^2)\mathbf{\Phi}^2\right] - \lambda^2 Z_\lambda v\mathbf{\Phi}(\mathbf{\Phi}^2+v^2) \\ &- \frac{1}{4}\lambda^2 Z_\lambda(\mathbf{\Phi}^4+v^4) - \frac{1}{4}Z_\omega\mathbf{F}^{\mu\nu}\mathbf{F}_{\mu\nu} + c(\mathbf{\Phi}+v) \end{aligned} \tag{12}$$

$$\mathfrak{L}_{source} = \bar{\eta}\mathbf{\Psi} + \bar{\mathbf{\Psi}}\eta + J_\phi\mathbf{\Phi} + J^\mu\mathbf{W}_\mu \tag{13}$$

or, equivalently

$$\begin{aligned} \mathfrak{L}_{QHD-I} =& \bar{\mathbf{\Psi}}\left(i\gamma^\mu\partial_\mu - M\right)\mathbf{\Psi} + \frac{1}{2}\left[(\partial_\mu\mathbf{\Phi})^2 - m^2\mathbf{\Phi}^2\right] \\ &- g\bar{\mathbf{\Psi}}\mathbf{\Phi}\mathbf{\Psi} + g_\omega\bar{\mathbf{\Psi}}\gamma^\mu(\omega_\mu+\mathbf{W}_\mu)\mathbf{\Psi} - \frac{1}{2}m_W^2(\omega_\mu+\mathbf{W}_\mu)^2 \\ &- \frac{1}{4}\lambda^2 v^2 - \frac{1}{4}(\mathbf{\Phi}^4+v^4) - \frac{1}{4}\mathbf{F}^{\mu\nu}\mathbf{F}_{\mu\nu} \\ &+ \frac{1}{2}\left[\delta_\phi(\partial_\mu\mathbf{\Phi})^2 - \delta m^2\mathbf{\Phi}^2\right] + \bar{\mathbf{\Psi}}\left[i\delta_\psi\gamma^\mu\partial_\mu - \delta M\right]\mathbf{\Psi} \\ &- g\delta_g\bar{\mathbf{\Psi}}\mathbf{\Phi}\mathbf{\Psi} - \frac{1}{4}\lambda^2\delta_\lambda v^2 - \frac{1}{4}\delta_\lambda(\mathbf{\Phi}^4+v^4) - \frac{1}{4}\delta_\omega\mathbf{F}^{\mu\nu}\mathbf{F}_{\mu\nu} \\ &- g_\omega\delta_{g_\omega}\bar{\mathbf{\Psi}}\gamma^\mu(\omega_\mu+\mathbf{W}_\mu)\mathbf{\Psi} - \frac{1}{2}\delta_{\mu\nu}(\omega_\mu+\mathbf{W}_\mu)^2 \\ &+ \bar{\eta}\mathbf{\Psi} + \bar{\mathbf{\Psi}} + J_\phi\mathbf{\Phi} + J^\mu\mathbf{W}_\mu + c(\mathbf{\Phi}+v), \end{aligned} \tag{14}$$

where g, g_ω and λ are the renormalized Yukawa couplings, and the terms

$$\delta_\psi = Z_\psi - 1, \quad \delta_\phi = Z_\phi - 1, \quad \delta_\omega = Z_\omega - 1, \tag{15}$$

$$\delta M = Z_\psi M_N - M, \ \ \delta m^2 = Z_\phi m_\phi^2 - m^2, \ \ \delta m_W^2 = Z_\omega m_\omega^2 - m_W^2, \tag{16}$$

$$\delta_g = Z_g - 1, \quad \delta_{g_\omega} = Z_{g_\omega} - 1, \quad \delta_\lambda = Z_\lambda - 1, \tag{17}$$

$$Z_g = 1 + \frac{\delta M}{M}, \ \ Z_{g_\omega} = Z_\psi \left(1 + \frac{\delta m_\omega^2}{m_\omega^2}\right), \ \ Z_\lambda = 1 + \frac{\delta m^2}{m^2}. \tag{18}$$

The renormalization conditions for the self - energies

$$\Sigma(k) = \bar{\Sigma}(k) + \delta M - \gamma_k \bar{k}^\mu (Z_\psi - 1) = \bar{\Sigma}(k) + \delta M - \delta_\psi k, \tag{19}$$

$$\Pi(k) = \bar{\Pi}(k) + \delta m^2 - k^2 (Z_\phi - 1) = \bar{\Pi}(k) + \delta m^2 - \delta_\phi k^2, \tag{20}$$

$$\begin{aligned} \Pi_{\mu\nu}(k) =& \bar{\Pi}_{\mu\nu}(k) - g_{\mu\nu}\delta m_W^2 - (k_\mu k_\nu - k^2 g_{\mu\nu})(Z_\omega - 1) \\ =& \bar{\Pi}_{\mu\nu}(k) - g_{\mu\nu}\delta m_W^2 - \delta_\omega (k_\mu k_\nu - k^2 g_{\mu\nu}), \end{aligned} \tag{21}$$

where $\bar{\Sigma}, \bar{\Pi}, \bar{\Pi}_{\mu\nu}$ are "unrenormalized" self-energies the (spinor) fermion, scalar meson and vector meson, respectively, and in (18) we introduced

$$\bar{k}^\mu = k^\mu - g_\omega \frac{Z_{g_\omega}}{Z_\psi} \mathbf{W}^\mu. \tag{22}$$

The renormalization conditions are imposed on the self - energies as follows

$$\Sigma(\not{k} = \mu_N) = 0, \qquad \frac{\partial \Sigma}{\partial \not{k}}(\not{k} = \mu_N) = 0, \tag{23}$$

$$\Pi(k^2 = \mu_\phi^2) = 0, \qquad \frac{\partial \Pi}{\partial k^2}(k^2 = \mu_\phi^2) = 0, \tag{24}$$

$$\Pi_{\mu\nu}(k^2 = \mu_\omega^2) = 0, \qquad \frac{\partial \Pi_{\mu\nu}}{\partial k^2}(k^2 = \mu_\omega^2) = 0, \tag{25}$$

here μ_N, μ_ϕ and μ_ω are the renormalization points.

The set of Dyson equations for propagators take the form

$$S = S_o + S_o \Sigma S, \tag{26}$$

$$G = G_o + G_o \Pi G, \tag{27}$$

$$D_{\mu\nu} = D_o^{\mu\nu} + D_{o\lambda}^{\mu} \Pi^{\lambda\rho} D_\rho^\nu, \tag{28}$$

where

$$k_\mu D^{\mu\nu}(k) = \frac{k^v}{m_W^2 + \delta m_W^2}, \tag{29}$$

$$k_\mu \Pi^{\mu\nu}(k) = - k^v \delta m_W^2, \quad k_\mu \bar{\Pi}^{\mu\nu}(k) = 0 \tag{30}$$

2.2. The equation of motion for scalar and vector mesons

From the Lagrangian (1), one gets

$$\frac{\delta \mathfrak{L}}{\delta \mathbf{\Phi}} = \left[Z_\phi \Box + (m_\phi^2 + \delta m_\phi^2)\right] \mathbf{\Phi} = J_\phi = \vec{j} - i S_\phi, \tag{31}$$

where

$$j_\phi = \frac{\delta \mathfrak{L}_{CT}}{\delta \mathbf{\Phi}} = c, \tag{32}$$

$$iS_\phi = \frac{\delta \mathfrak{L}_L}{\delta \mathbf{\Phi}} = gZ_g \bar{\mathbf{\Psi}}\mathbf{\Psi} + \lambda^2 Z_\lambda (2v\mathbf{\Phi}^2 + v^2 + \mathbf{\Phi}^3), \tag{33}$$

and

$$\frac{\delta \mathfrak{L}}{\delta \omega_\mu} = Z_\omega \Box + (m_\omega^2 + \delta m_\omega^2)\omega^\mu = J_\omega^\mu = -j_\omega^\mu + iS_\omega^\mu, \tag{34}$$

where

$$j_\omega^\mu = W^\mu (m_W^2 + \delta m_W^2) = \frac{\delta \mathfrak{L}_{CT}}{\delta W^\mu}, \tag{35}$$

$$iS_\omega^\mu = \frac{\delta \mathfrak{L}_L}{\delta W^\mu} = g_\omega Z_{g_\omega} \bar{\Psi}\gamma_\mu \Psi. \tag{36}$$

Eqs.(31) and (34) determine "unrenormalized" source J_ϕ and J_ω^μ. The conditions

$$\langle F |\mathbf{\Phi}| F\rangle = 0, \qquad \langle F |\omega^\mu| F\rangle = 0 \tag{37}$$

imply

$$J_\phi = j_\phi - iS_\phi = 0, \qquad -J_\omega^\mu = j_\omega^\mu - iS_\omega^\mu = 0 \tag{38}$$

or, equivalently

$$-J_\omega^\mu = W^\mu (m_W^2 + \delta m_W^2) - \frac{g_\omega Z_{g_\omega}}{Z_\psi} J_B^\mu = 0, \tag{39}$$

where $J_B^\mu = (\rho, j_B)$ is the baryon current in the medium

$$W^\mu = \frac{g_\omega Z_\psi \left(1 + \frac{\delta m_W^2}{m_W^2}\right)}{Z_\psi \left(m_W^2 + \delta m_W^2\right)} J_B^\mu = \frac{g_\omega}{m_\omega^2} J_B^\mu \tag{40}$$

2.3. The free real-time Green's functions in momentum space

1. Scalar propagators in the real-time formalism are defined as

$$G_o^{++}(\mathbf{k}, t, t') = G_o^{>}(\mathbf{k}, t, t')\theta(t - t') + G_o^{<}(\mathbf{k}, t, t')\theta(t' - t), \tag{41}$$

$$G_o^{--}(\mathbf{k}, t, t') = G_o^{>}(\mathbf{k}, t, t')\theta(t' - t) + G_o^{<}(\mathbf{k}, t, t')\theta(t - t'), \tag{42}$$

$$G_o^{-+}(\mathbf{k}, t, t') = G_o^{>}(\mathbf{k}, t, t'), \tag{43}$$

$$G_o^{+-}(\mathbf{k}, t, t') = G_o^{<}(\mathbf{k}, t, t'), \tag{44}$$

$$\begin{aligned} G_o^{>}(\mathbf{k}, t, t') &= i \int d^3x e^{-i\mathbf{kx}} \left\langle \Phi(\mathbf{x}, t)\Phi(\mathbf{0}, t')\right\rangle \\ &= \frac{i}{2\omega_k} \left\{ [1 + n_B(\omega_k)]\, e^{-i\omega_k(t-t')} + n_B(\omega_k) e^{i\omega_k(t-t')} \right\} \end{aligned} \tag{45}$$

$$\begin{aligned} G_o^{<}(\mathbf{k}, t, t') &= i \int d^3x e^{-i\mathbf{kx}} \left\langle \Phi(\mathbf{0}, t')\Phi(\mathbf{x}, t)\right\rangle \\ &= \frac{i}{2\omega_k} \left\{ n_B(\omega_k) e^{-i\omega_k(t-t')} + [1 + n_B(\omega_k)]\, e^{i\omega_k(t-t')} \right\} \end{aligned} \tag{46}$$

where $\omega_k = \sqrt{\mathbf{k}^2 + m^2}$, and $n_B(\omega) = \frac{1}{e^{\beta\omega} - 1}$ is the Bose - Einstein distribution.

2. Fermion propagators (zero fermion chemical potential) are defined by

$$S_o^{++}(\mathbf{k},t,t') = S_o^{>}(\mathbf{k},t,t')\theta(t-t') + S_o^{<}(\mathbf{k},t,t')\theta(t'-t), \tag{47}$$

$$S_o^{--}(\mathbf{k},t,t') = S_o^{>}(\mathbf{k},t,t')\theta(t'-t) + S_o^{<}(\mathbf{k},t,t')\theta(t-t'), \tag{48}$$

$$S_o^{-+}(\mathbf{k},t,t') = S_o^{>}(\mathbf{k},t,t'), \tag{49}$$

$$S_o^{+-}(\mathbf{k},t,t') = S_o^{<}(\mathbf{k},t,t'), \tag{50}$$

$$\begin{aligned} S_o^{>}(\mathbf{k},t,t') &= -i\int d^3x e^{-i\mathbf{kx}} \left\langle \Psi(\mathbf{x},t)\bar{\Psi}(\mathbf{0},t')\right\rangle \\ &= -\frac{i}{2\bar{\omega}_k}\Big\{ (\gamma^o\bar{\omega}_k - \gamma\mathbf{k} + M)\left[1-n_F(\bar{\omega}_k)\right] e^{-i\bar{\omega}_k(t-t')} \\ &\quad + (\gamma^o\bar{\omega}_k + \gamma\mathbf{k} - M)\, n_F(\bar{\omega}_k) e^{i\bar{\omega}_k(t-t')}\Big\} \end{aligned} \tag{51}$$

$$\begin{aligned} S_o^{<}(\mathbf{k},t,t') &= i\int d^3x e^{-i\mathbf{kx}} \left\langle \bar{\Psi}(\mathbf{0},t')\Psi(\mathbf{x},t)\right\rangle \\ &= \frac{i}{2\bar{\omega}_k}\Big\{ (\gamma^o\bar{\omega}_k + \gamma\mathbf{k} - M)\, n_F(\bar{\omega}_k) e^{-i\bar{\omega}_k(t-t')} \\ &\quad + (\gamma^o\bar{\omega}_k + \gamma\mathbf{k} - M)\left[1-n_F(\bar{\omega}_k)\right] e^{i\bar{\omega}_k(t-t')}\Big\} \end{aligned} \tag{52}$$

where $\bar{\omega}_k = \sqrt{\mathbf{k}^2 + M^2}$, and $n_F(\omega) = \frac{1}{e^{\beta\omega}+1}$ is the Fermi - Dirac distribution.

These free propagators given in Eqs.(45), (46) and (51), (52) are thermal because the initial state in chosen to be in thermal equilibrium and the interaction in assumed to be turned on adiabatically.

3. The renormalized effective diracequation

We aim our effort at the relaxation of inhomogeneous fermion mean field $\psi(\mathbf{x},t) = \langle \mathbf{\Psi}(\mathbf{x},t)\rangle$ induced by external source that is adiabatically switched on at $t=-\infty$. At usually, the fermion field is shifted by

$$\Psi^{\pm}(\mathbf{x},t) = \psi(\mathbf{x},t) \pm \varphi^{\pm}(\mathbf{x},t), \tag{53}$$

with $\langle\varphi^{\pm}(\mathbf{x},t)\rangle = 0$.

3.1. The Initial Value Problem The effective real time Dirac equation for the mean field of momentum $\mathbf{k}$ reads

$$\begin{aligned} &\left[(i\gamma^o\partial_t - \gamma\mathbf{k} - M) + \delta_\psi\left(i\gamma^o\partial_t - \gamma\mathbf{k}\right) - \delta_M\right]\Psi(\mathbf{k},t) \\ &- \int_{-\infty}^{t} dt'\Sigma(\mathbf{k},t-t')\Psi(\mathbf{k},t') = -\eta(\mathbf{k},t), \end{aligned} \tag{54}$$

where $\partial_t \equiv \frac{\partial}{\partial t}$, $\Sigma(\mathbf{k},t-t')$ is the retarded fermion self - energy and

$$\Psi(\mathbf{k},t) \equiv \int d^3x e^{-i\mathbf{kx}}\Psi(\mathbf{x},t) \tag{55}$$

The source is taken to be switched on adiabatically from $t=-\infty$ and switched off at $t=0$ to provide the initial condition

$$\Psi(\mathbf{k},t=0) = \Psi(\mathbf{k},0); \qquad \Psi(\mathbf{k},t<0) = 0. \tag{56}$$

Introducing an auxiliary quantity $\chi(\mathbf{k}, t-t')$ defined as

$$\Sigma(\mathbf{k}, t-t') = \partial_{t'}\chi(\mathbf{k}, t-t'), \tag{57}$$

and imposing $\eta(\mathbf{k}, t>0) = 0$, we obtain the following equation of motion for $t > 0$

$$\begin{aligned}\Big[(i\gamma^o\partial_t - \gamma\mathbf{k} - M) + \delta_\psi\,(i\gamma^o\partial_t - \gamma\mathbf{k}) - \chi(\mathbf{k},0) - \delta_M\Big]\Psi_k(t)\\ + \int_0^t dt'\chi_k(t-t')\Psi_k(t') = 0.\end{aligned} \tag{58}$$

This equation of motion can be solved by Laplace transform as befits an initial value problem. The Laplace transformed equation of motion is given by

$$\begin{aligned}&[i\gamma^o s - \gamma\mathbf{k} - M + \delta_\psi\,(i\gamma^o s - \gamma\mathbf{k}) - \delta_M - \chi(\mathbf{k},0) + s\tilde{\chi}(s,\mathbf{k})]\,\tilde{\Psi}(s,\mathbf{k})\\ &= [i\gamma^o + i\delta_\psi\gamma^o + \tilde{\chi}(s,\mathbf{k})]\,\Psi(\mathbf{k},0),\end{aligned} \tag{59}$$

where

$$\tilde{\Psi}(s,\mathbf{k}) \equiv \int_0^\infty dt e^{-st}\Psi(\mathbf{k},t);\;\; \tilde{\chi}(s,\mathbf{k}) \equiv \int_0^\infty dt e^{-st}\chi(\mathbf{k},t), \tag{60}$$

with $Res > 0$.
We can write $\chi(\mathbf{k}, t-t')$ as

$$\chi(\mathbf{k},t-t') = i\gamma^o\chi^{(0)}(\mathbf{k},t-t') + \gamma\mathbf{k}\chi^{(1)}(\mathbf{k},t-t') + \chi^{(2)}(\mathbf{k},t-t'). \tag{61}$$

A straightforward calculation leads to the ultraviolet divergences

$$\chi^{(1)}(\mathbf{k},0) \simeq \frac{g^2}{16\pi^2}ln\frac{\Lambda}{\mu}, \chi^{(2)}(\mathbf{k},0) \simeq -\frac{g^2 M}{8\pi^2}ln\frac{\Lambda}{\mu}, \tilde{\chi}^{(0)}(s,\mathbf{k}) \simeq \frac{g^2}{16\pi^2}ln\frac{\Lambda}{\mu}, \tag{62}$$

where $\tilde{\chi}^{(i)}(s,\mathbf{k}), (i = 0,1,2)$ are the Laplace transform of $\chi^{(i)}(\mathbf{k},t)$, Λ is an ultraviolet momentum cutoff, μ is an arbitrary renormalization scale.

From Eq.(59), one gets

$$\begin{aligned}&\Big[i\gamma^o s - \gamma\mathbf{k} - M + \delta_\psi i\gamma^o s - \delta_\psi\gamma\mathbf{k} - \delta_M - \gamma\mathbf{k}\frac{g^2}{16\pi^2}ln\frac{\Lambda}{\mu} - \frac{g^2 M}{8\pi^2}ln\frac{\Lambda}{\mu} + s\tilde{\chi}(s,\mathbf{k})\Big]\tilde{\Psi}(s,\mathbf{k})\\ &= [i\gamma^o + i\delta_\psi\gamma^o + \tilde{\chi}(s,\mathbf{k})]\,\Psi(\mathbf{k},0).\end{aligned} \tag{63}$$

The counter-terms δ_ψ and δ_M are chosen as

$$\delta_\psi = -\frac{g^2}{16\pi^2}ln\frac{\Lambda}{\mu} + \mathit{finite}, \delta_M = \frac{g^2 M}{8\pi^2}ln\frac{\Lambda}{\mu} + \mathit{finite}, \tag{64}$$

and the components of the self-energy are rendered finite

$$\begin{aligned}\chi(\mathbf{k},0) + \gamma\mathbf{k}\delta_\psi + \delta_M &= \mathit{finite},\\ \tilde{\chi}(s,\mathbf{k}) + i\gamma^o\delta_\psi &= \mathit{finite}.\end{aligned} \tag{65}$$

3.2. Renormalized effective Dirac equation

Hence, we obtain the renormalized effective Dirac equation in the medium and the corresponding initial value problem for the fermion mean field

$$\Big[i\gamma^o s - \gamma\mathbf{k} - M - \tilde{\Sigma}(s,\mathbf{k})\Big]\,\tilde{\Psi}(s,\mathbf{k}) = [i\gamma^o + \tilde{\chi}(s,\mathbf{k})]\,\Psi(\mathbf{k},0). \tag{66}$$

Compare with (59), it is easy to derive the form of $\tilde{\Sigma}(s,\mathbf{k})$

$$\tilde{\Sigma}(s,\mathbf{k}) = \chi(\mathbf{k},0) - s\tilde{\chi}(s,\mathbf{k}), \tag{67}$$

This is the Laplace transform of the renormalized retarded fermion self-energy , which can be written in its most general form

$$\tilde{\Sigma}(s,\mathbf{k}) = i\gamma^o s\tilde{\varepsilon}^{(0)}(s,\mathbf{k}) + \gamma\mathbf{k}\tilde{\varepsilon}^{(1)}(s,\mathbf{k}) + M\tilde{\varepsilon}^{(2)}(s,\mathbf{k}). \tag{68}$$

The solution of Eq. (66) is given by

$$\tilde{\Psi}(s,\mathbf{k}) = \frac{1}{s}\left\{1 + S(s,\mathbf{k})\left[\gamma\mathbf{k} + M + \tilde{\Sigma}(0,\mathbf{k})\right]\right\}\Psi(\mathbf{k},0), \tag{69}$$

where $S(s,\mathbf{k})$ is the fermion propagator in terms of the Laplace variable s

$$\begin{aligned} S(s,\mathbf{k}) &= \left[i\gamma^o s - \gamma\mathbf{k} - M - \tilde{\Sigma}(s,\mathbf{k})\right]^{-1} \\ &= -\frac{i\gamma^o s\left\{1 - \tilde{\varepsilon}^{(0)}(s,\mathbf{k}) - \gamma\mathbf{k}\left[1 + \tilde{\varepsilon}^{(1)}(s,\mathbf{k})\right] + M\left[1 + \tilde{\varepsilon}^{(2)}(s,\mathbf{k})\right]\right\}}{s^2\left[1 - \tilde{\varepsilon}^{(0)}(s,\mathbf{k})\right]^2 + k^2\left[1 + \tilde{\varepsilon}^{(1)}(s,\mathbf{k})\right]^2 + M^2\left[1 + \tilde{\varepsilon}^{(0)}(s,\mathbf{k})\right]^2}. \end{aligned} \tag{70}$$

The square of the denominator in eq.(70) is being

$$\det\left[i\gamma^o s - \gamma\mathbf{k} - M - \tilde{\Sigma}(s,\mathbf{k})\right]. \tag{71}$$

The real-time evolution of $\Psi(\mathbf{k},t)$ is obtained by performing this inverse Laplace transform in the complex s-plane along contour parallel to the imaginary axis.

The denominator can be rewritten in the form $\left(\omega^2 - \omega_k^2 - P(\omega,\mathbf{k})\right)$, where

$$\begin{aligned} P(\omega,\mathbf{k}) &= -2\left[\omega^2\varepsilon^{(0)}(\omega,\mathbf{k}) + k^2\varepsilon^{(1)}(\omega,\mathbf{k}) + M^2\varepsilon^{(2)}(\omega,\mathbf{k})\right] \\ &= \frac{1}{2}Tr\left[\left(\gamma^0\omega - \gamma\mathbf{k} + M\right)\Sigma(\omega,\mathbf{k})\right]. \end{aligned} \tag{72}$$

It is just the lowest order term of effective self-energy imaginary part of $P(\omega,\mathbf{k})$ evaluated on the fermion mass shell.

4. Quantum kinetic equation for fermion in QHD - I

Let us denote the distribution function for fermion of momentum $\mathbf{k}$ and spin s by $\bar{n}_{s,k}(t)$. Since for a fixed spin component the matrix elements for transition probabilities are rather cumbersome, we study the spin - averaged fermion distribution function as $\bar{n}_k(t) = \frac{1}{2}\sum_s \bar{n}_{s,k}(t)$.

For a small departure from thermal equilibrium, one can approximate $\left\langle \mathbf{\Phi}^2\right\rangle$ and $\left\langle \mathbf{W}^2\right\rangle$ by their thermal equilibrium values

$$\left\langle \Phi^2\right\rangle = \int \frac{d^3k}{(2\pi)^3\omega_k} n_B(\omega_k), \tag{73}$$

$$\left\langle W^2\right\rangle = \delta_{\mu\nu}\left\langle W^\mu W^\nu\right\rangle = \delta_{\mu\nu}\int \frac{d^3k}{(2\pi)^3\omega_k} n_B^{\mu\nu}(\omega_k). \tag{74}$$

To two - loop order, the Feynman diagrams that contribute to the kinetic equation is shown in Fig.1

The kinetic equation can be derived directly basing on [6]

$$\begin{aligned}\frac{d}{dt}\bar{n}_k(t) = \pi g^2 \int \frac{d^3q}{(2\pi)^3}&\frac{\bar{\omega}_k\bar{\omega}_q - \mathbf{kq} - M^2}{2\bar{\omega}_q\bar{\omega}_k\omega_p}\delta(\bar{\omega}_k + \bar{\omega}_q - \omega_p)\\ \times\Big\{ &[n_B(\omega_p)\,(1-\bar{n}_k(t))\,(1-\bar{n}_q(t)) - (1+n_B(\omega_p))\,\bar{n}_k(t)\bar{n}_q(t)]\\ &+ \left[\delta_{\mu\nu}n_B^{\mu\nu}(\omega_p)\,(1-\bar{n}_k(t))\,(1-\bar{n}_q(t)) - \left(1+\delta_{\mu\nu}n_B^{\mu\nu}(\omega_p)\right)\bar{n}_k(t)\bar{n}_q(t)\right]\Big\},\end{aligned} \tag{75}$$

where $\mathbf{p} = \mathbf{k} + \mathbf{q}$.

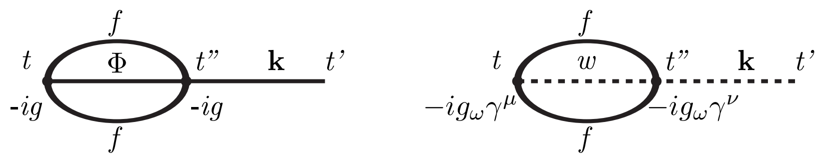

Fig. 1. The Feynman diagrams contribute to the kinetic equation for fermion's interaction up to two loop order. The bold solid line is the fermion propagator S, the only solid line is the scalar propagator G and the dashed line is the omega propagator $D_{\mu\nu}$

It is easy to find that the above equation has an equilibrium solution given by $\bar{n}_k(t) = n_F(\bar{\omega}_k)$ for all momentum $\mathbf{k}$

$$\bar{n}_F(t) = n_F(\bar{\omega}_k) + \delta\bar{n}_k(t), \tag{76}$$

where $\frac{\delta\bar{n}_k(t)}{n_F(\bar{\omega}_k)} << 1$. Retaining linear terms in $\delta\bar{n}_k(t)$ from Eq.(75), one obtains the equation for $\delta\bar{n}_k(t)$

$$\frac{d}{dt}\delta\bar{n}_k(t) = -\Gamma(k)\bar{n}_k(t), \tag{77}$$

where $\Gamma(k)$ is the interaction rate, whose inverse characterizes the time scale for the fermion distribution to approach equilibrium [11]

$$\begin{aligned}\Gamma(k) =& \pi g^2 \int \frac{d^3q}{(2\pi)^3}\frac{\bar{\omega}_k\bar{\omega}_q - \mathbf{kq} - M^2}{2\bar{\omega}_q\bar{\omega}_k\omega_p}\delta(\bar{\omega}_k + \bar{\omega}_q - \omega_p)\\ &\times\left\{[n_B(\omega_p) + n_F(\bar{\omega}_q)] + \left[\delta_{\mu\nu}n_B^{\mu\nu}(\omega_p) + n_F(\bar{\omega}_q)\right]\right\}\\ =& \frac{g^2m^2T}{16\pi k\bar{\omega}_k}\left(1-\frac{4M^2}{m^2}\right) ln\frac{1-e^{-\beta(\bar{\omega}_q+\bar{\omega}_k)}}{1+e^{-\beta\bar{\omega}_q}}\Bigg|_{q=q^-}^{q=q^+}\\ &+\frac{g^2m_w^2T}{16\pi k\bar{\omega}_k}\left(1-\frac{4M^2}{m_W^2}\right) ln\frac{1-e^{-\beta(\bar{\omega}_q+\bar{\omega}_k)}}{1+e^{-\beta\bar{\omega}_q}}\Bigg|_{q=q_W^-}^{q=q_W^+},\end{aligned} \tag{78}$$

where

$$q^{\pm} = \frac{m^2}{2M^2}\left|k\left(1-\frac{2M^2}{m^2}\right) \pm \sqrt{(k^2+M^2)\left(1-\frac{4M^2}{m^2}\right)}\right|, \tag{79}$$

$$q_W^{\pm} = \frac{m_W^2}{2M^2}\left|k\left(1-\frac{2M^2}{m_W^2}\right) \pm \sqrt{(k^2+M^2)\left(1-\frac{4M^2}{m_W^2}\right)}\right|, \tag{80}$$

with $q \in (q^-, q^+)$, $q_W \in \left(q_W^-, q_W^+\right)$ are the support of $\delta\left(\bar{\omega}_k - \omega_p + \bar{\omega}_q\right)$ for fixed $\mathbf{k}$.

The kinetic analysis is implemented directly in real-time and clearly establishes the relation between the interaction rate in the relaxation time approximation and the damping rate of the mean field.

5. Discussion and conclusion

In the above mentioned sections the real time formalism was used to study the fermion propagator in the matter modeled by the QHD-I model. It could eventually be used in other problems and non equilibrium processes in the medium of finite density and temperature.

We have presented and solved the renormalized effective Dirac equation by Laplace transform. The formulation of the initial value problem yields unambiguous separation of the vacuum and in-medium effects. We obtained the kinetic equation for fermion in the QHD-I model, including the fermion's interaction with the neutral scalar and vector mesons. The fermion distribution in non equilibrium is investigated. It is proportional to the interaction rate, whose inverse characterizes the time scalar for the fermion distribution to approach equilibrium. Our next paper is intended to be devoted to the quantum kinetic equation for scalar, pseudoscalar and vector meson in the QHD.II model.

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