

Analysis and identification of multi-variate random pressure fields using covariance and spectral proper transformations

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Abstract. This paper will present applications of the Proper Transformations based on both cross spectral matrix and covariance matrix branches to analysis and identification of multi-variate random pressure fields. The random pressure fields are determined due to the physical measurements on some typical rectangular models in the wind tunnel tests. The significant roles of the first covariance mode associated with the first principal coordinates as well as of the first spectral eigenvalue and associated spectral mode are clarified in reconstructing the random pressure fields and identifying the hidden physical phenomena inside this pressure fields.

Keywords: random pressure fields, proper orthogonal decomposition, proper transformations.

1. Introduction

Aerodynamic phenomena of structures due to the atmospheric wind flows are generated by spatial distribution and correlation of random fluctuating pressure field on surface of structural section. The fluctuating pressure field can be represented as spatially-correlated multi-variate random processes. Understanding and knowledge of the random pressure field and its distribution is possible to interpret mechanisms of excitations, identification and response of aerodynamic phenomena happening on structure. Due to the nature of random field, however, the fluctuating pressure field is considered as superposition from some causes and excitation of dominant physical phenomena. It is logical thinking to decompose the total pressure field by sums of independently partial pressure fields, which can be related to a particular mechanism of excitation and certain physical phenomena.

The Proper Orthogonal Decomposition (POD) was developed by Loeve 1945 and Karhunen 1946, thus also known as the Karhunen-Loeve decomposition, was firstly applied for analyzing random fields by Lumley 1970 [1], Berkooz et al. 1993 [2] as a stochastic decomposition to decouple multi-variate random turbulent fields. The POD also has been widely used for many fields such as analysis, simulation of random fields (including the random pressure field), numerical analysis, dynamic system

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identification, dynamic response and so on. Several literatures presented the POD's application to decompose the spatially-correlated and multi-variate random pressure fields into uncorrelated random processes and basic orthogonal vectors (also called as POD modes or shape-functions). The POD has been branched by either covariance matrix-based or spectral matrix-based proper orthogonal decompositions and associated proper transformations, which depend on how to build up a basic matrix from either zero-time-lag covariance or cross spectral matrices of the multi-variate random processes.

Up to now, analyses of the random pressure fields almost have based on the covariance matrix-branched POD due to its straightforward in computation and interpretation. Some authors used the POD to analyze random pressure field and to find out relation between POD modes and physical phenomena (eg., [3-8]). Bienkiewicz et al. 1995 [3] used the POD analysis of mean and fluctuating pressure fields around low-rise building directly measured due to turbulent flows. A linkage between pattern of the pressure distribution and POD modes, especially first two modes was discussed and interpreted, in which the 1st mode was compatible to the pattern of the fluctuating pressure distribution, whereas the 2nd mode similar to the mean pressure pattern. Holmes et al. 1997 [4], however, reviewed that that no consistent linkages between physical phenomena and POD mode due to series of physical measurements and POD analyses of pressure fields in low-rise buildings. Effect of pressure tap positions on the same measured pressure area (uniform and non-uniform arrangements) on POD modes studied by Jeong et al. 2000 [5], by which POD modes observed differently in two cases. Kikuchi et al. 1997 [6] applied the POD to pressure field of tall buildings, then fluctuating pressure field was reconstructed due to only few dominant POD modes, used to estimate aerodynamic forces and corresponding responses. Tamura et al. 1997&1999 [7-8] indicated distortion and wrong interpretation of POD modes due to presence of mean pressure data in the analyzed pressure field. It is argued that the POD is appropriate tool to reveal physical phenomena on from experimental data where correspondence between the POD modes and physical causes from the fluctuating pressure field. However, some others discussed that interpretation from POD modes is aprioristic and arbitrary based from previous knowledge of system behavior and response. Application to the pressure field analyses based on spectral matrix-branched POD is rare due to its troublesome. Recently, De Grenet and Ricciardelli 2004 [9] pioneered in using the spectral matrix-based POD to study the pressure field on squared cylinders, however, it has troublesome and difficulties in interpreting theses results.

In this paper, the POD based spectral and covariance matrices of the random field will be presented. Both covariance-based and spectral-based POD modes of the wind-induced fluctuating pressure field have been analyzed to find out possible relationships between the POD modes and physical phenomena, characteristics of bluff body flows as well. Surface pressure field has been determined through physical measurements on some typical rectangular models with side ratios of $B/D=1$ and $B/D=5$ in the wind tunnel tests.

2. Proper orthogonal decomposition

2.1. Definition

The POD is optimum approximation of random field. The main idea of the POD is to find out a set of orthogonal basic vectors which can expand a multi-variate random process into a sum of products of these basic orthogonal vectors and single-variant uncorrelated random processes. Let consider the unsteady surface pressure field is expressed as:

$$P(v, t) = \bar{p}(v) + p(v, t) \quad (1)$$

where $P(v, t)$: unsteady pressure; $\bar{p}(v)$: mean pressure; $p(v, t)$: fluctuating pressure; v : dimensional variables ($v=x; y; z$). Fluctuating pressure field $p(v, t)$ is usually represented as N-variate random process with zero mean containing sub-processes at N points in the field: $p(v, t) = \{p_1(v, t), p_2(v, t), \dots, p_N(v, t)\}$. This field can be expressed as following approximation:

$$p(v, t) = a(t)^T \Phi(v) = \sum_i a_i(t) \phi_i(v) \quad (2)$$

where $a_i(t)$: i-th principal coordinate as uni-variate zero-time random processes $E[a_i(t)] = 0$; $\phi_i(v)$: i-th basic orthogonal vector $\phi_i(v)^T \phi_j(v) = \delta_{ij}$ (δ_{ij} : Kronecker delta); $a(t) = \{a_1(t), a_2(t), \dots, a_N(t)\}$, $\Phi(v) = [\phi_1(v), \phi_2(v), \dots, \phi_N(v)]$.

In mathematical expression of optimality is to find out space function $\Phi(v)$ to maximize the projection of random field $p(v, t)$ onto this space function, suitably normalized due to the mean square basis [1]:

$$\text{Max} \frac{\langle | (p(v, t) \otimes \Phi(v)) |^2 \rangle}{\|\Phi(v)\|^2} \quad (3)$$

where $(\otimes), \langle \cdot \rangle, ||, ||$ denote to inner product, expectation, absolute and Euler squared norm operators, respectively.

2.2. Covariance matrix-based proper orthogonal decomposition

The optimality in (3) can expand under the form of equality:

$$\int_{L_v} R(v, v', t) \Phi(v') dv' = \lambda \Phi(v) \quad (4)$$

where $R(v, v', t)$: covariance value as spatial correlation between two points v, v' in the random field; λ : weighted coefficient.

Thus solution of space function $\Phi(v)$ can be determined as the eigen problem as follows:

$$R_p(v, t) \Phi(v) = \Lambda \Phi(v) \quad (5)$$

where $R_p(v, t)$: covariance matrix of fluctuating pressure sub-processes in field, by which is defined as $R_p(v, t) = [R_{ij}(v, t)]_{N \times N}$, $R_{ij}(v, t) = E[p(v_i, t)^T p(v_j, t)]$, $p(v_i, n)$: pressure sub-process at position v_i ; Λ : diagonal eigenvalue matrix $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$; $\Phi(v)$: eigenvector matrix (also called POD modes).

The random fluctuating pressure field can be reconstructed due to limited number of the lowest POD modes:

$$p(v, t) = a(t) \Phi(v) \approx \sum_{i=1}^{\tilde{N}} a_i(t) \phi_i(v), \quad \tilde{N} < N \quad (6)$$

In Eq.(6), the principal coordinate can be computed from measured data:

$$a(t) = \Phi(v)^{-1} p_0(v, t) = \Phi(v)^T p_0(v, t) \quad (7)$$

where $p_0(v, t)$: measured data or observations.

In the covariance matrix-branched POD, some characteristics can be deducted from the eigen problems as follows:

$$\Phi(v)^T \Phi(v) = I; \Phi(v)^T R_p(v, t) \Phi(v) = \Lambda \quad (8a)$$

$$E[a_i(t)^T a_j(t)] = \lambda_i \delta_{ij}; \sigma_{p_i}^2 = E[p(v_i, t)^2] = \lambda_i; R_{p_i p_j} \approx \sigma_i \sigma_j \sum_{k=1}^{\tilde{N}} \lambda_k \phi_{ik} \phi_{jk} \quad (8b)$$

In order to estimate the contribution percentage of i-th covariance POD mode on total random field, one is based on either proportion of eigenvalues as follows:

$$E_{\phi_i} = \frac{\lambda_i}{\sum_{i=1}^N \lambda_i} \% \quad (9)$$

Afterward this procedure is applied for analysis and identification of the random pressure field.

2.3. Spectral matrix-based proper orthogonal decomposition

Similar to the covariance matrix-branched POD, cross spectral matrix can be defined from the fluctuating pressure field as $S_p(v, f) = [S_{ij}(v, f)]_{N \times N}$, $S_{ij}(v, f) = E[\hat{p}(v_i, f)^T \hat{p}(v_j, f)]$, where $\hat{p}(v_i, f), \hat{p}(v_j, f)$: Fourier transforms of the fluctuating pressure sub-processes $p(v_i, t), p(v_j, t)$ at space v_i, v_j ; f: frequency variables.

Then spectral space function $\Phi(v, f)$ (depending on frequency) can be determined based upon the eigen problem of the cross spectral matrix $S_p(v, f)$ of the fluctuating pressure field $p(v, t)$ as:

$$S_p(v, f) \Phi(v, f) = \Lambda(f) \Phi(v, f) \quad (10)$$

where $\Lambda(f), \Phi(v, f)$: spectral eigenvalue and eigenvector matrices, $\Lambda(f) = \text{diag}[\lambda(f), \lambda(f), \dots, \lambda(f)]$, $\Phi(v, f) = [\phi_1(v, f), \phi_2(v, f), \dots, \phi_N(v, f)]$ (also known as spectral POD modes).

The random fluctuating pressure field can be reconstructed due to limited number of the lowest spectral POD modes:

$$\hat{p}(v, f) = \hat{a}(f) \Phi(v, f) \approx \sum_{i=1}^{\tilde{N}} \hat{a}_i(f) \phi_i(v, f), \tilde{N} < N \quad (11a)$$

$$S_p(v, f) = \Phi(v, f) \Lambda(f) \Phi^{*T}(v, f) \approx \sum_{i=1}^{\tilde{N}} \phi_i(v, f) \lambda_i(f) \phi_i^{*T}(v, f), \tilde{N} < N \quad (11b)$$

where $\hat{p}(v, f), \hat{S}_p(v, f)$: Fourier transform and power spectrum of reconstructed pressure field $p(v, t)$; *, T: complex conjugate and transpose operations; $\hat{a}(f)$: spectral principal coordinates as Fourier transforms of uncorrelated single-variate random processes which can be computed from measured data:

$$\hat{a}(f) = \Phi(v, f)^{-1} \hat{p}_0(v, f) = \Phi(v, f)^T \int_{-\infty}^{\infty} p_0(v, t) e^{i2\pi ft} dt \quad (12)$$

where $\hat{p}_0(v, f)$: Fourier transform of measured data or observations $p_0(v, t)$.

Some characteristics can be deduced from the spectral matrix-branched POD and the eigen problems as follows:

$$\Phi^{*T}(v, f) \Phi(v, f) = I; \Phi^{*T}(v, f) S_p(v, f) \Phi(v, f) = \Lambda(f) \quad (13)$$

Energy contribution of i-th spectral POD mode on total field energy can be determined as proportion of spectral eigenvalues on limited frequency range as follows:

$$E_{\phi_i(f)} = \frac{\sum_k^{f_{cutoff}} \lambda_i(f_k)}{\sum_{i=1}^N \sum_{k=1}^{f_{cutoff}} \lambda_i(f_k)} \% \quad (14)$$

This spectral matrix-branched procedure will be applied for analysis and identification of pressure field.

3. Wind tunnel experiments

Physical pressure measurements were carried out in the Kyoto University's open-circuit wind tunnel. Three typical rectangular models with slender ratios $B/D=1$, $B/D=1$ (with Splitter Plate), $B/D=5$ were used. Artificial turbulent flows were generated in the wind tunnel at mean wind velocities 3m/s (case1), 6m/s (case 2) and 9m/s (case 3), corresponding to intensities of turbulence were $I_u=11.46\%$, $I_w=11.23\%$; $I_u=10.54\%$, $I_w=9.28\%$; $I_u=9.52\%$, $I_w=6.65\%$, respectively. Pressure measurement holes were arranged inside, in chordwise direction and on one surface of models in which model $B/D=1$ labeled pressure positions from 1 to 10, whereas model $B/D=5$ from 1 to 19. Unsteady surface pressures were simultaneously measured by the multi-channel pressure measurement system (ZOC23 system: Z (Zero), O (Operation), C (Calibration)). Electric signals were filtered by 100Hz low-pass filters (E3201, NF Design Block Co., Ltd.) before passed through A/D converter (Thinknet DF3422, Pavac Co., Ltd.) with sampling frequency at 1000Hz in 100 seconds.

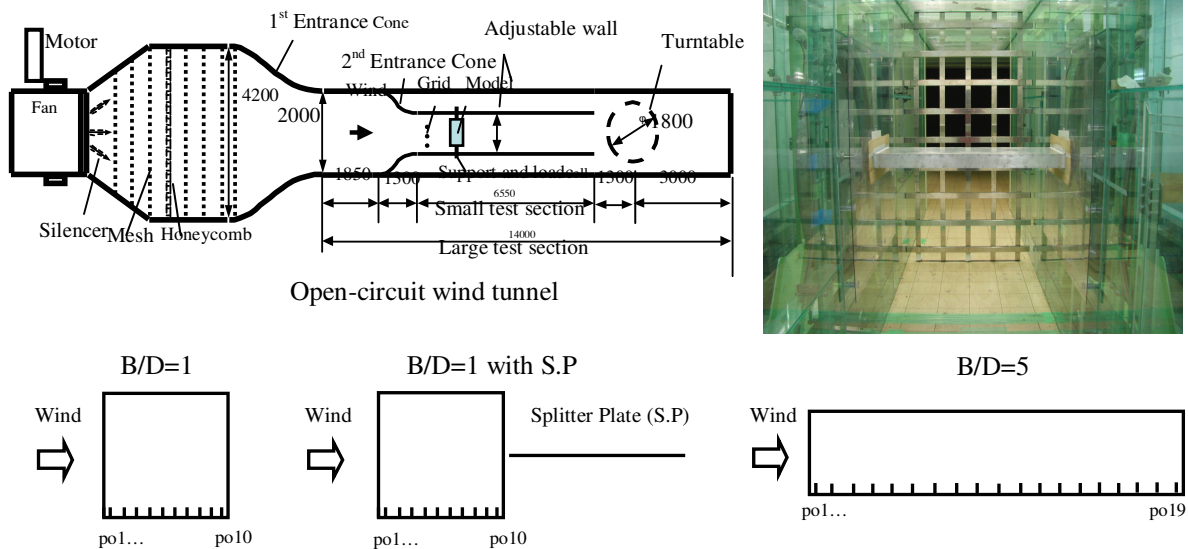


Fig. 1. Wind tunnel configuration, experimental set-ups and experimental models.

Flow around models due to interaction between ongoing flow and model section is usually known as the bluff body flow, which characterized by formation of separated and reattached flows with separation bubble and formation of vortex shedding as well. It can be predicted from the past knowledge that model $B/D=1$ is favorable for formation of Karman vortex shedding, where model

$B/D=5$ is typical for formation of separated and reattached flows on model surface. The splitter plate was added to model $B/D=1$ to suppress effect of Karman vortex.

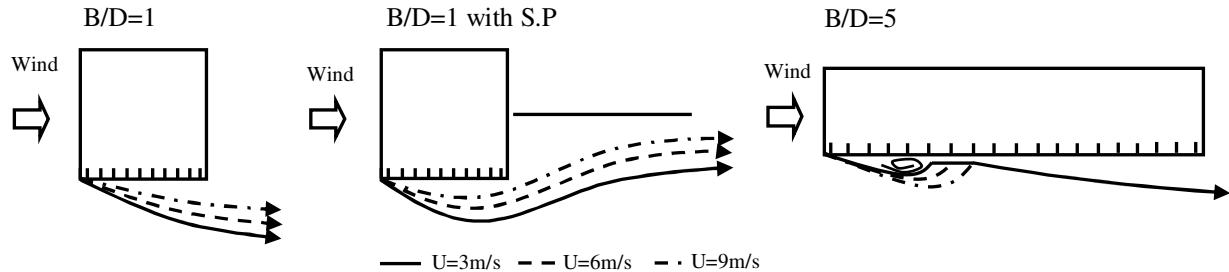


Fig. 2. Bluff body flow patterns around experimental models.

The bluff body flow patterns around three experimental models can be predicted as shown above in Figure 2 (bluff-body flows on one surface are drawn).

4. Surface pressure distribution and bluff body flow pattern

Mean and root-mean-square fluctuating pressure coefficients have been normalized by dynamic pressure component from measured unsteady pressure data as follows:

$$C_{p,mean} = \bar{p} / (0.5\rho U^2); C_{p,rms} = \sigma_p / (0.5\rho U^2) \quad (15)$$

where $0.5\rho U^2$: dynamic pressure; $\bar{p}$: mean pressure; σ_p : standard deviation of unsteady pressure.

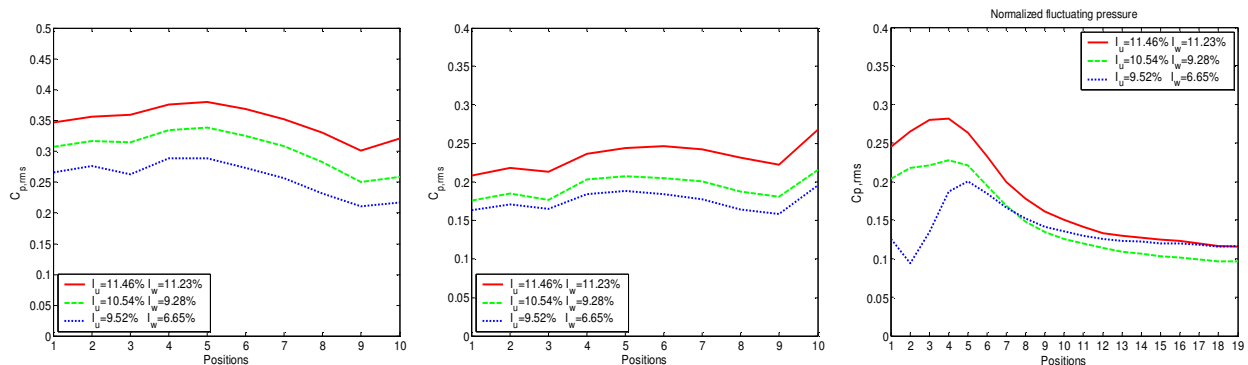


Fig. 3. Normalized fluctuating pressure distribution on chordwise positions.

Figure 3 shows the chordwise distributions of normalized fluctuating pressures on models at three turbulent flow conditions. As can be seen that the fluctuating pressure distributes steadily on whole surface of models $B/D=1$ but distributes dominantly on the leading region of the model $B/D=5$. The fluctuating pressures, furthermore, reduce with respect to decrease of intensities of turbulence.

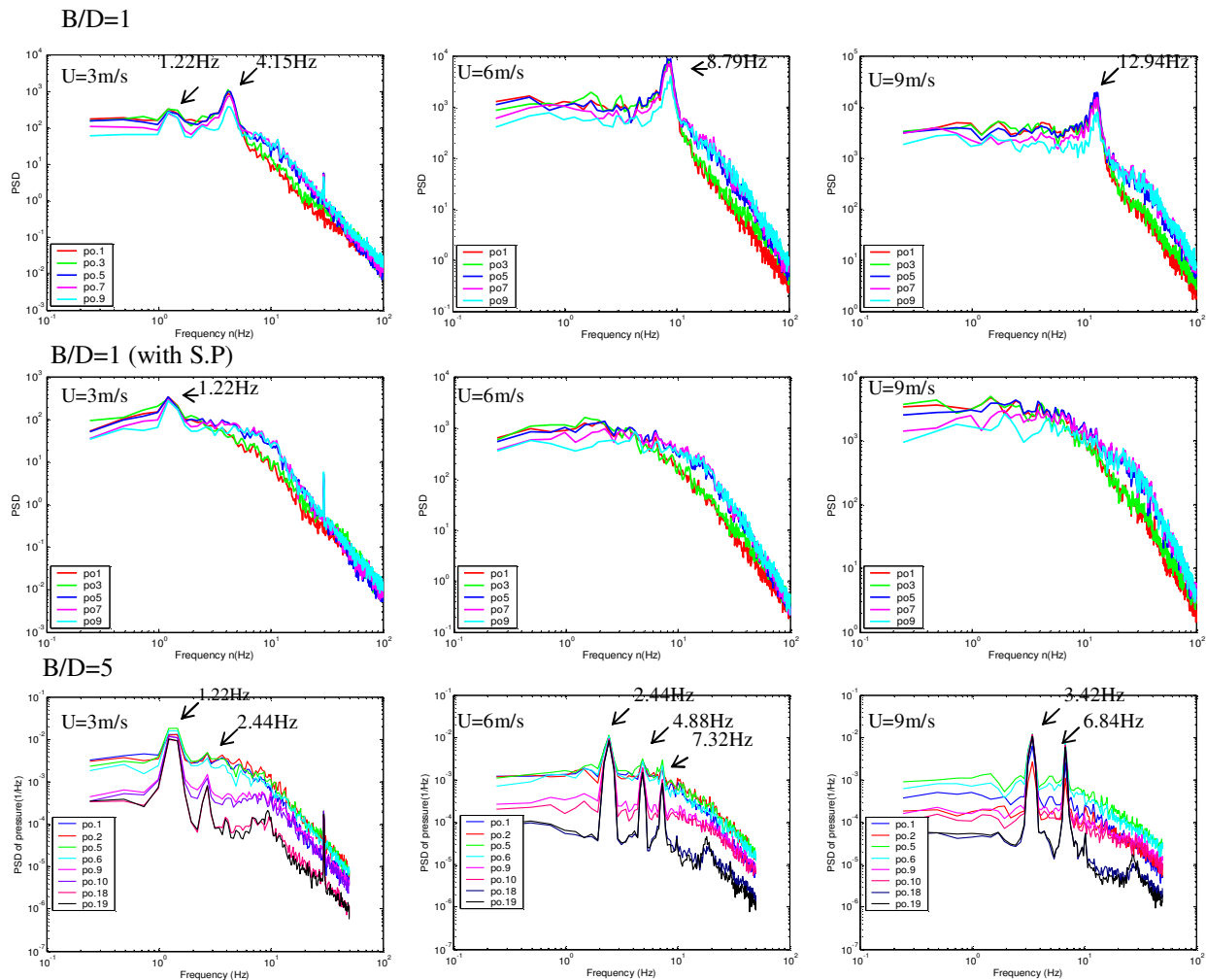


Fig. 4. Power spectra of fluctuating pressures at some chordwise positions.

Figure 4 indicates power spectra of the fluctuating pressures at some chordwise positions with three models and turbulent conditions. As can be seen with the model B/D=1 (without splitter plate) that peaked frequencies are observed at 4.15Hz, 8.79Hz and 12.94Hz respective to the three turbulent flows. It is explained that the Karman vortex formed and shed at the wake of model. Shedding frequency depends on the Strouhal number (S_t) of cross section, moreover, the Strouhal number can be determined $St=0.1285$. In case B/D=1 with splitter plate, no peaked frequency is observed, it also means that no Karman vortex occurred and the splitter plate has suppressed effect of the Karman vortex. In case of the model B/D=5, spectral peaks are also observed at frequencies 1.22Hz and 2.44Hz ($U=3\text{m/s}$); at 2.44Hz, 4.88Hz, 7.32Hz (case 2); at 3.42Hz and 6.84Hz (case 3). It is predicted that the bluff body flow is separated and reattached one. Reattachment points are at roughly positions 6, 7, 8 with respect to an increase of mean velocities. It is supposed that the observed spectral peaks are induced by rolled-up vortices shed away at reattachment points toward trailing edge. This agrees well with findings presented in the literatures of Hiller and Cherry, 1981 and Cherry et al., 1984 which were proposed empirical formula to estimate frequency of rolled-up vortices shedding at reattachment point depending on mean velocity and length of separation bubble.

5. Results and discussion

5.1. Analysis on covariance matrix branch

Eigenvalues and eigenvectors (covariance pressure modes) have been determined from covariance matrix of chordwise fluctuating pressures. Figure 5 shows first four covariance modes along chordwise positions at the flow case 1 of $U=3\text{m/s}$ (two other cases are similar and not be interpreted here for sake of brevity). It is noted that all first covariance modes look alike to the fluctuating pressure distributions.

Energy contribution of the lowest covariance modes, estimated following Eq.(9) is given in Table 1. Obviously, the first covariance mode contributes dominantly to system, energy contribution here calculates following the Eq.(9). The first covariance modes contribute 76.92%, 65.29%, 43.77% to total energy at the flow case 1 corresponding to models $B/D=1$ with and without the splitter plate and model $B/D=5$, respectively in the flow case 1. If first two covariance modes are taken into account, the energy of these modes holds up to 90.19%, 86.26%, 65.79% of total energy. It is noted that the first covariance mode in the model $B/D=5$ holds energy contribution of only 43.77% to compare with that of 76.92%, 65.29% in the other models of $B/D=1$. This can be explained due to complexity of bluff body flow around the model $B/D=5$ to reduce a role of the first covariance mode.

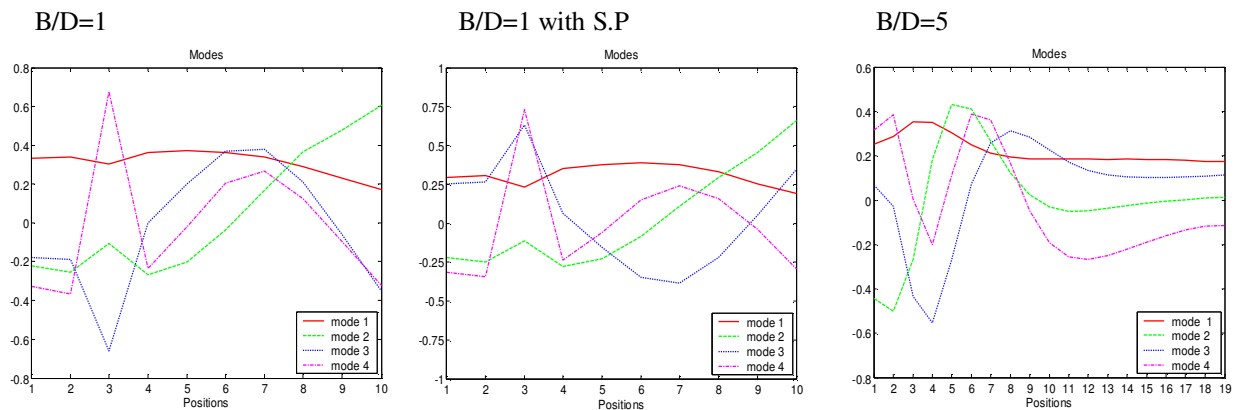


Fig. 5. First four covariance pressure modes of experimental models.

Table 1. Energy contribution of covariance pressure modes (%)

Mode	B/D=1			B/D=1 with S.P			B/D=5		
	3m/s	6m/s	9m/s	3m/s	6m/s	9m/s	3m/s	6m/s	9m/s
1	76.92	77.46	75.36	65.29	62.79	63.30	43.77	44.86	65.9
2	13.27	13.25	14.41	20.97	22.61	22.08	22.02	23.14	13.29
3	4.69	4.23	4.62	6.14	6.29	6.10	15.18	15.14	9.48
4	2.87	2.86	3.17	4.04	4.32	4.41	5.98	5.68	3.40
5	1.27	1.32	1.45	1.99	2.28	2.45	4.76	4.11	2.79

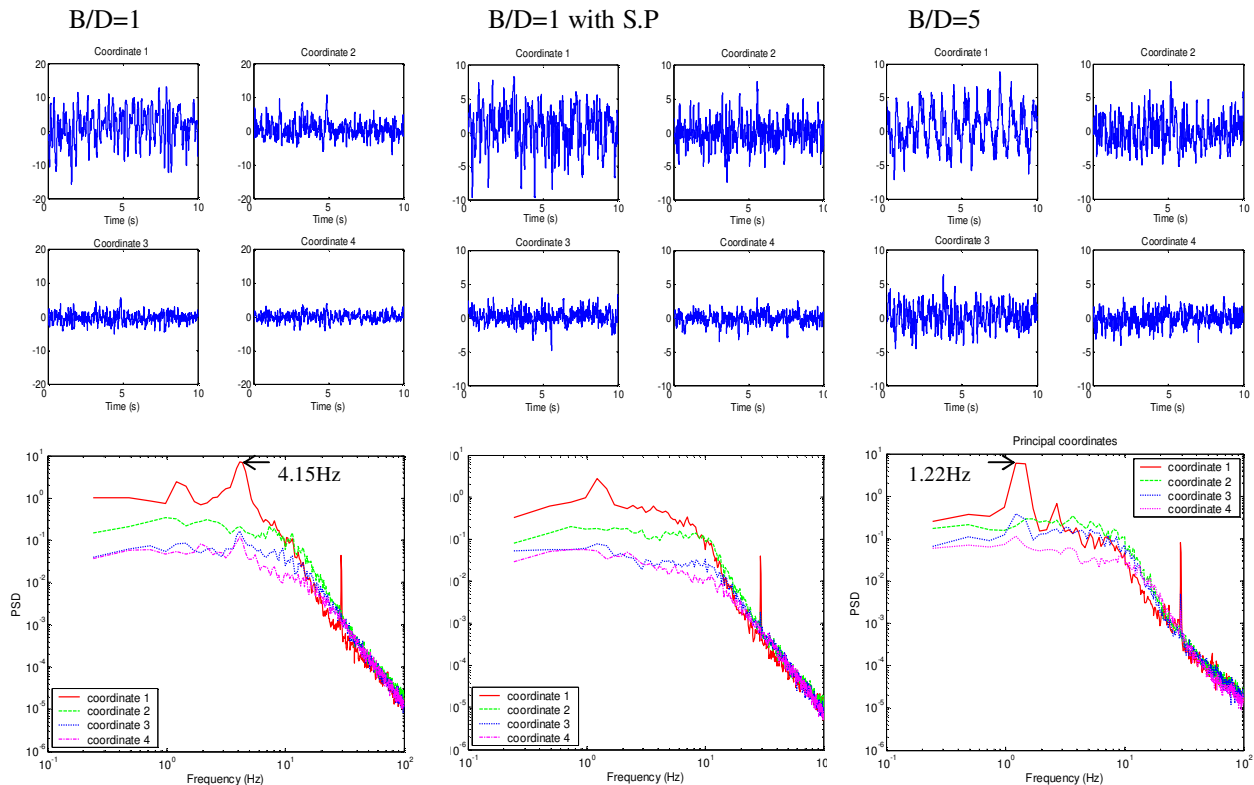


Fig. 6. First four principal coordinates and their power spectral densities.

Uncorrelated principal coordinates associated with the covariance pressure modes has been calculated from the measured pressure data, as first four principal coordinates of three models at the flow case 1 and their corresponding power spectra are shown in Figure 6. It is noteworthy that first coordinates not only dominate in the power spectrum but contain frequency characteristics of the random pressure field, whereas the other coordinates do not contain these frequencies. Thus, the first covariance pressure modes and associated principal coordinate will play very important role in the identification of random pressure field due to their dominant energy contribution and frequency containing of hidden physical events of system.

5.2. Analysis on spectral matrix branch

Frequency dependant eigenvalues and eigenvectors (spectral modes) are obtained from the cross spectral matrix of the observed pressure field. Figure 7 shows first five spectral eigenvalues on frequency band 0÷50Hz at the flow case 1 ($U=3\text{m/s}$). As can be seen from Figure 7, all first spectral eigenvalues from three models exhibit much dominantly than others, especially theses first eigenvalues also contain all frequency peaks of the pressure field, whereas others do not hold theses peaks. This finding means in these investigations that the first spectral mode can represent for hidden characteristics of the pressure fields, concretely here the first mode contains frequency of any physical phenomenon happening on models.

Energy contributions of spectral pressure modes are expressed in Table 2. Similar to the covariance pressure modes, the first spectral pressure modes contain dominantly the system energy of

the unsteady pressure fields, for example, the first pressure mode contribute 86.04%, 81.30%, 74.77%, respectively to the three experimental models at the flow case 1 ($U=3\text{m/s}$). In the cases of two modes combined, the first two pressure modes contribute almost 94.12%, 91.45%, 87.45% on the total energy, respectively. It is also the same as the covariance matrix branch that the first spectral mode contributes 74.77% to the energy in the model $B/D=5$, whereas it holds 86.04% and 81.30% in two other models of $B/D=1$. This might be also due to an influence of separating and reattachment flow on the modal surface, moreover, it might suggest that the more complicate the random pressure fields exhibit the less important the first mode contributes.

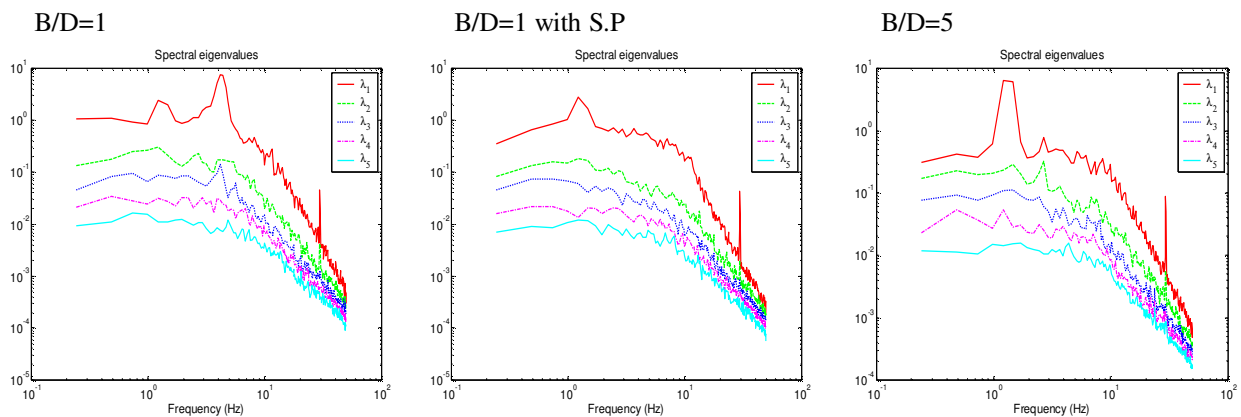


Fig. 7. First five spectral eigenvalues of experimental models.

Table 2. Energy contribution of spectral pressure modes (%)

Mode	B/D=1			B/D=1 with S.P			B/D=5		
	3m/s	6m/s	9m/s	3m/s	6m/s	9m/s	3m/s	6m/s	9m/s
1	86.04	85.84	83.02	81.30	77.48	77.88	74.77	73.59	83.93
2	8.08	8.08	9.92	10.15	12.36	11.98	12.68	14.03	7.69
3	3.28	3.20	3.68	4.44	5.14	5.00	5.68	5.56	3.57
4	1.40	1.62	1.94	2.05	2.63	2.70	2.75	2.86	1.86
5	0.64	0.72	0.81	1.09	1.28	1.34	1.44	1.45	1.06

In comparison on the energy contribution between the covariance modes and the spectral ones, as can be seen from Tables 1 and 2 that the first spectral mode contributes higher than the first covariance one. Concretely, the first spectral mode holds 94.12%, 91.45%, 87.45% comparing with 76.92%, 65.29%, 43.77% of the first covariance one in the three models of $B/D=1$, $B/D=1$ with splitter plate and $B/D=5$, respectively at the flow case 1 ($U=3\text{m/s}$), similarly, 83.02%, 77.88%, 83.93% to compare with 75.36%, 63.3%, 65.9% at flow case 3 ($U=9\text{m/s}$). It might suggest that the first spectral mode exhibits better than the first covariance one in the analysis, synthesis and identification of the random pressure fields.

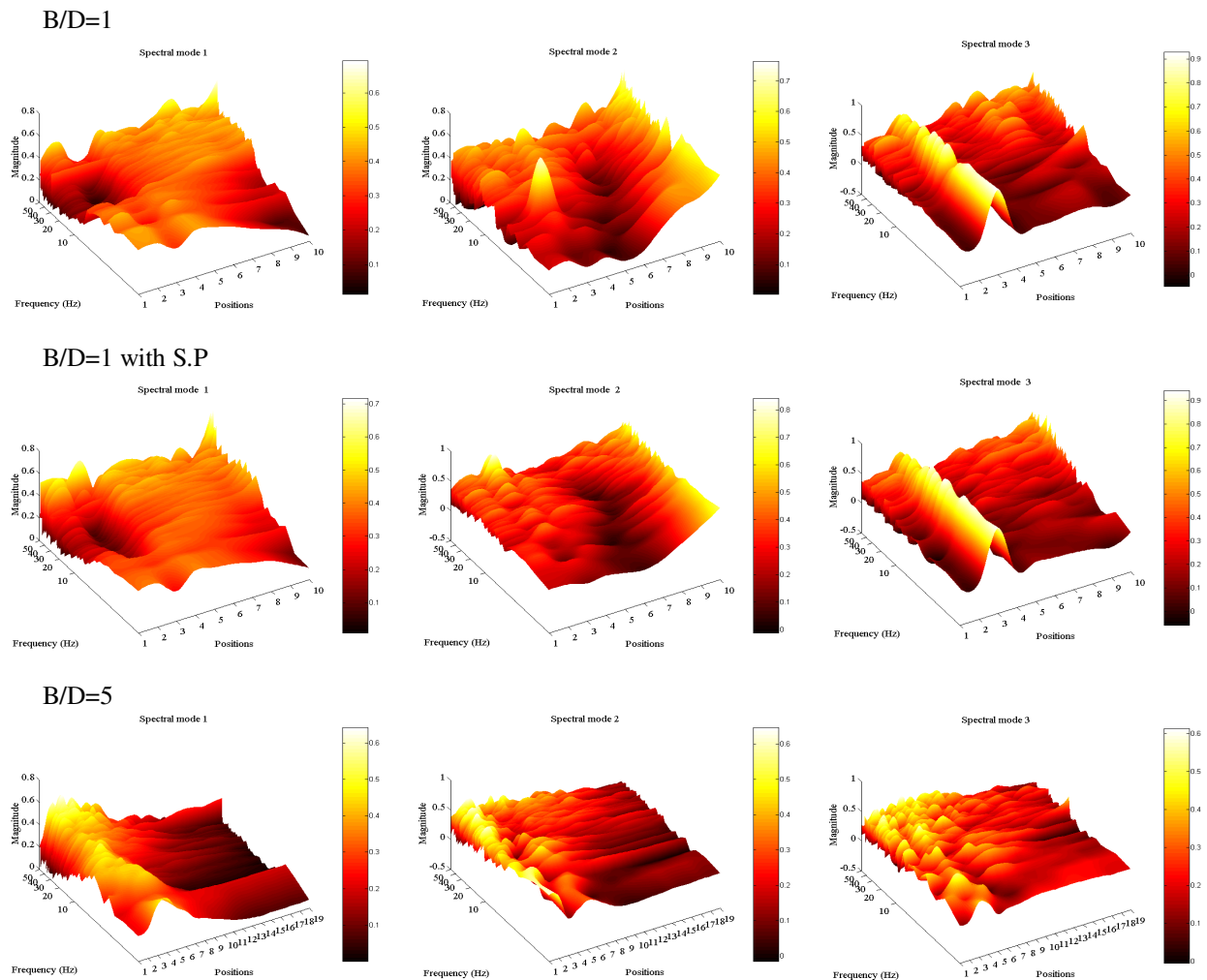


Fig. 8. First three spectral pressure modes of experimental models.

The first three spectral pressure modes of the chordwise fluctuating pressure fields of experimental models in the flow case 1 are shown at Figure 8 in frequency band 0-50Hz. It seems that more investigations must be needed to clarify the physical meaning of the spectral pressure modes as well as the linkage between the pressure modes and hidden events of the unsteady pressure fields.

6. Synthesis and identification of random pressure field

Firstly, effects of basic and cumulative covariance modes on the synthesis of the unsteady pressure fields, as well as role of the first covariance mode on the identification of these pressure fields will be verified and investigated. Figures 9 and Figure 10 show the pressure synthesis and the spectral pressure one at referred position 5 using individually basic covariance modes (1st mode, 2nd mode, 3rd mode and 4th mode), whereas Figure 11 indicated the cumulative covariance modes (1st mode and first 2 modes), respectively with verifying spectral synthesis of the covariance modes to original time series of pressures (as target), only position 5 and flow case 1 presented due to brevity.

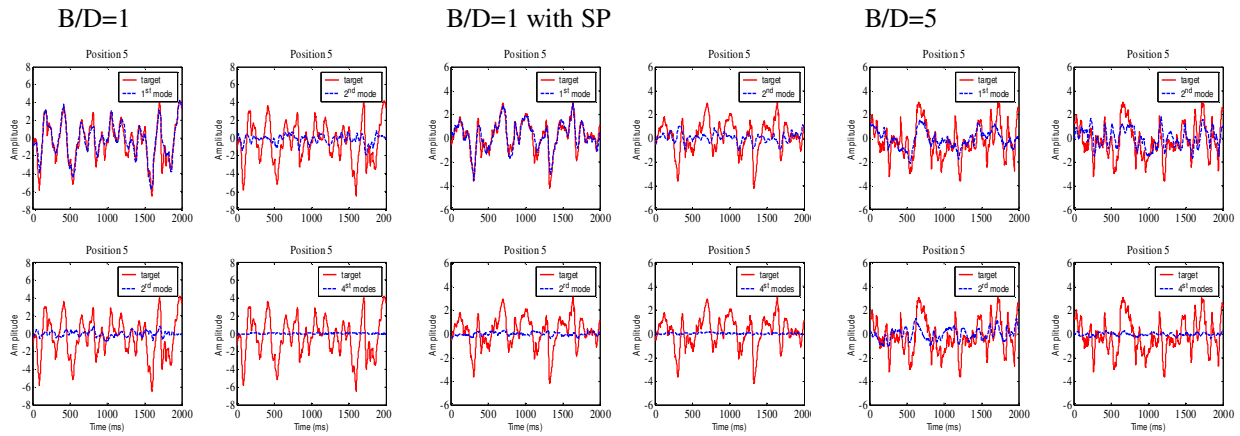


Fig. 9. Effect of basic covariance modes on pressure synthesis at referred position 5.

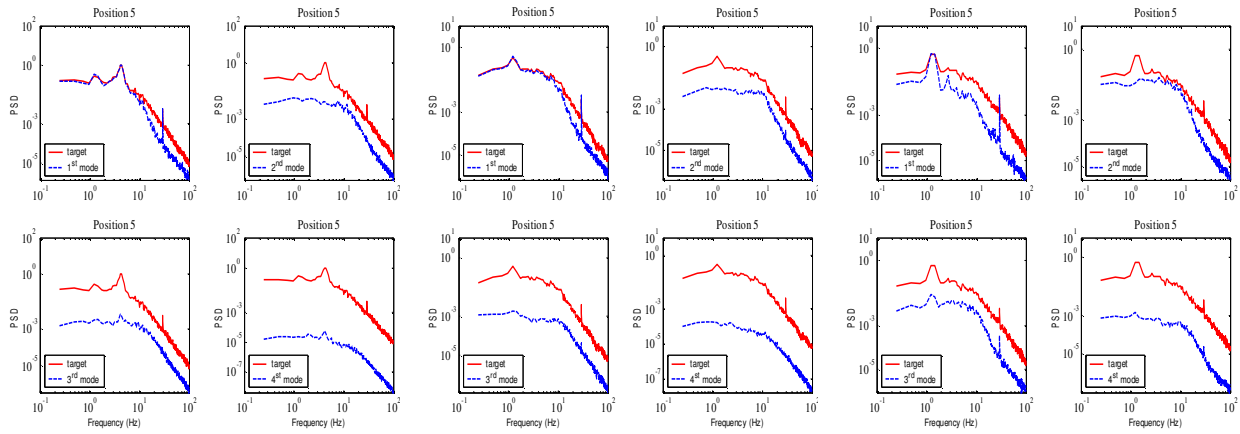


Fig. 10. Effect of basic covariance modes on spectral pressure synthesis at referred position 5.

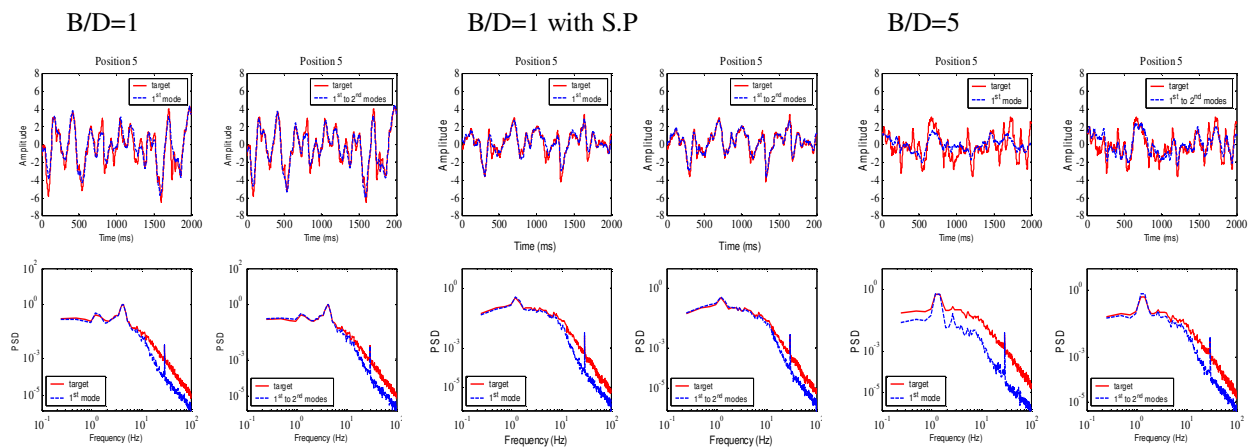


Fig. 11. Effect of cumulative covariance modes on pressure synthesis at referred position 5.

As can be seen from Figure 9, Figure 10 that reconstructed pressure time series using the first covariance pressure mode is similar to the original pressure, especially its containing of frequency peaks can be used to identify hidden characteristics and physical phenomena of the original pressure.

In comparison, reconstructed pressure portions using 2nd mode, 3rd mode, 4th mode are minor contributions to the original pressure, and these pressure portions do not contain the frequency peaks in the original pressure. Reconstructed pressure using the first mode, moreover, seems to be good agreement to the original pressure at low frequency range between 0-10Hz in models B/D=1, but it is notable in spectral difference between reconstructed pressure and original one at high frequency range in models B/D=1 and all frequency range (excepting at frequency peaks) in model B/D=5. It is argued that the first mode is enough for the reconstructed pressure at the low frequencies in models B/D=1, but more cumulative modes may be needed for the reconstructed pressure at the high frequencies. In the model B/D=5, moreover, the first mode can be used to identify the field, but it is not enough to reconstruct the original pressure, then more modes should be needed for the pressure reconstruction due to more complicate distribution of pressure field.

In the Figure 11, reconstructed pressure using the first mode and cumulative two modes and their PSD are presented, it can be seen that only the first mode is enough to reconstruct the original pressure in models B/D=1, the cumulative two modes are enough in model B/D=5.

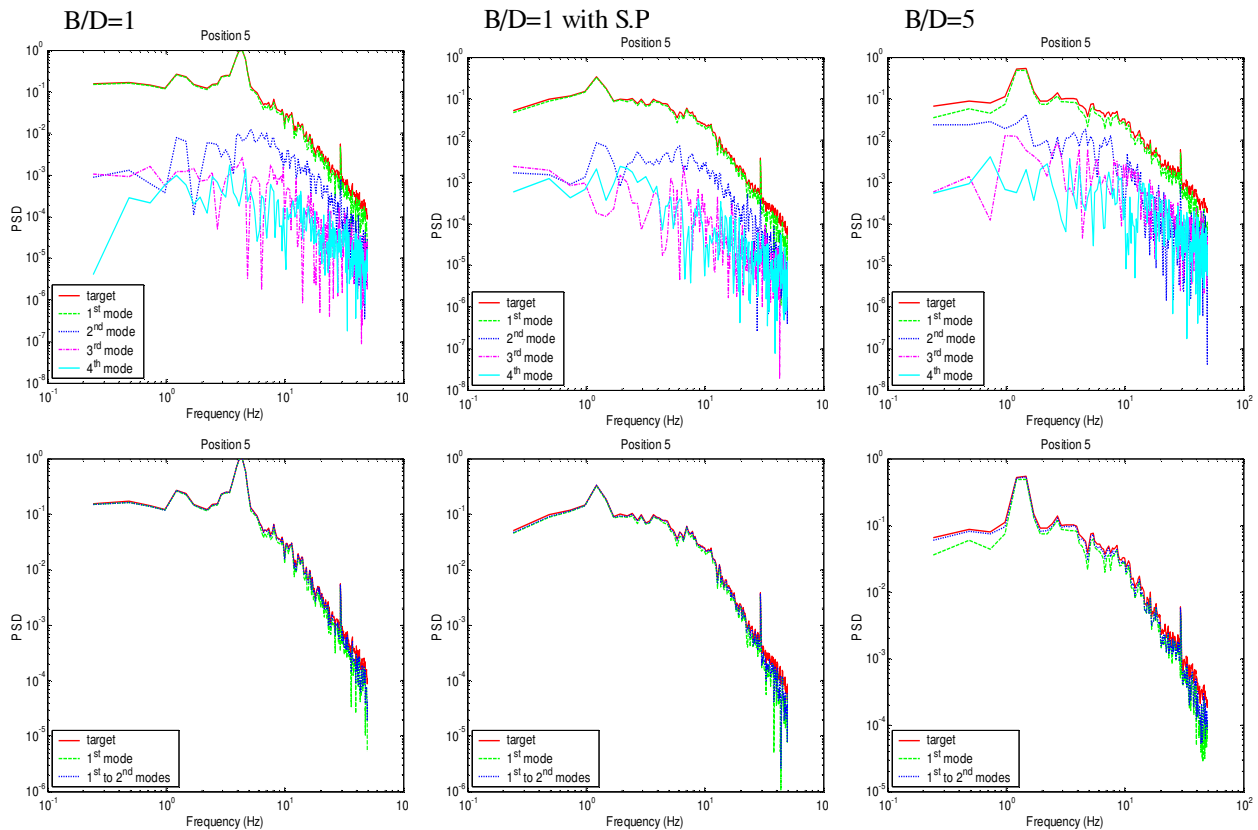


Fig. 12. Effects of basic and cumulative spectral modes on spectral synthesis of pressure position 5.

Secondly, effects of basic and cumulative spectral pressure modes on the synthesis of the unsteady pressure fields, as well as role of the first spectral mode on the identification of these pressure fields

will be investigated. Figure 12 above shows the effects of individual and cumulative spectral modes on the synthesis of auto spectra density of the pressure fields, here pressure at referred position 5 in the flow case 1 of $U=3\text{m/s}$ is used for demonstration. As can be seen in upper row that the first spectral mode only is accuracy enough to reconstruct and identify the original pressure in all three experimental models and whole frequency range. There are also good agreements between spectrum of the original pressure and reconstructed spectrum using the first mode and cumulative two modes.

7. Conclusion

Analysis and identification of the unsteady pressure fields measured on some typical rectangular sections using both the Covariance Proper Transformation in the time domain and the Spectral Proper Transformation in the frequency domain have been presented in this paper. So-called the covariance pressure modes and the spectral pressure ones have been orthogonally decomposed from the covariance matrix and the spectral one as the comprehensive descriptions of the unsteady pressure fields. Some conclusions can be pointed out as follows:

The first covariance pressure mode and the first spectral mode as well play very important role in analysis, synthesis and identification of the unsteady pressure fields. It contributes dominantly the system energy of the pressure fields as well as contains certain frequency peaks of possibly physical phenomena hidden in these pressure fields. Moreover, it seems that the first spectral pressure mode exhibits better than the first covariance one in the analysis, synthesis and identification of the unsteady pressure fields

In low frequency range, only the first mode (either the covariance pressure mode or the spectral pressure one) can reconstruct the unsteady pressure fields with enough accuracy, whereas more cumulative modes should be needed to reconstruct the unsteady pressure field in the cases of the high frequency range and of the complicated pressure distributions and flows as well. In other words, the more complicated the pressure field distributes and the bluff body flow behaviors, the less important the first mode contributes and the more cumulative modes are needed to reconstruct the pressure fields.

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